Hale School Physics 3B 2010

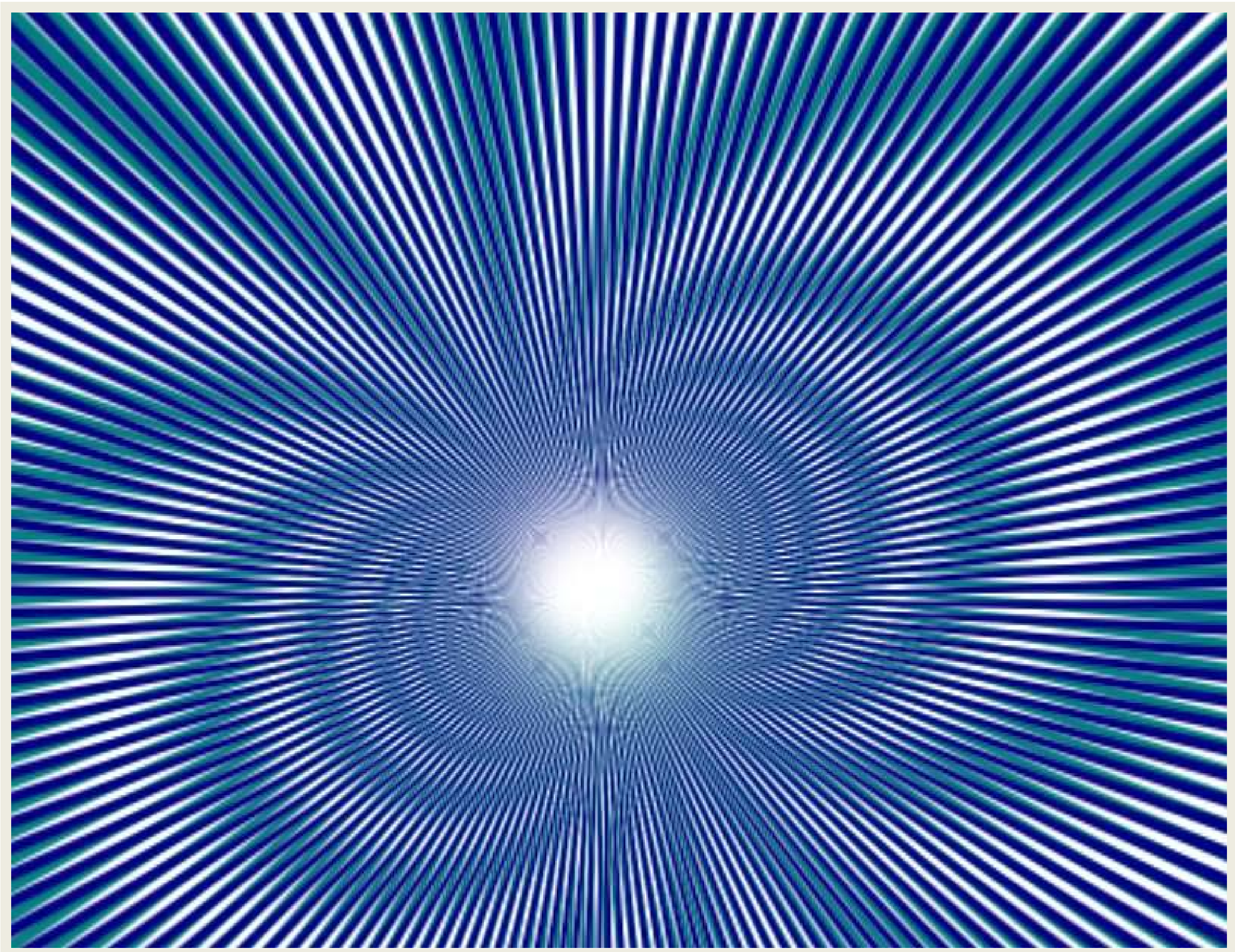
Wave Theory

Year 12 Study Notes

Name:

Teacher:

Set:



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The Nature of Waves

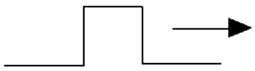
A wave is a means of transferring energy from one point to another through a medium, without transferring matter.

Some waves move through a medium causing particles in the medium to vibrate back and forth about fixed positions. There is no net (overall) movement of the medium. What is observed to move is the waveform. e.g. a sound travels through air by vibrating the air particles about fixed positions.

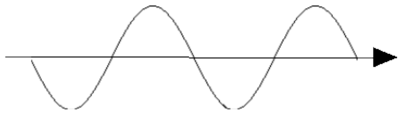
- A wave which moves from one point to another point is termed a progressive wave.

- A wave pulse is a short wave form that transmits a burst of energy through a medium.

- A wave train is a repeated wave form which transmits energy continuously through a medium.



Wave Pulse



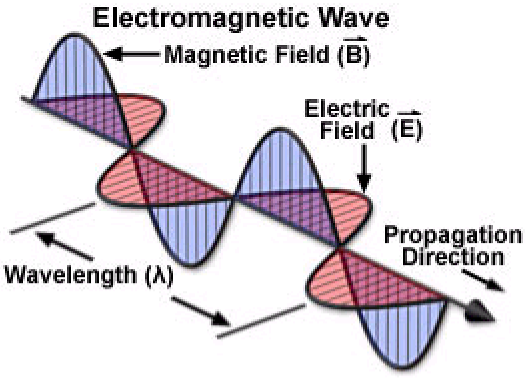
Wave Train

Wave Classification

1. Medium or No Medium

Mechanical waves require a physical medium (i.e. particle medium) for their propagation e.g. sound and water waves, and waves in strings.

Electromagnetic waves consist of oscillating transverse electric and magnetic fields which vibrate mutually at right angles to their direction of propagation.



These waves do not require a particle medium and may travel through a vacuum.

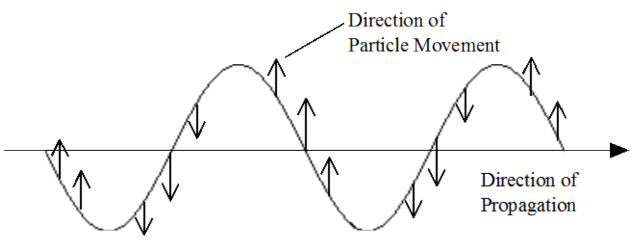
All electromagnetic waves travel at the speed of light 3.00×10^8 ms^{-1} in a vacuum.

2. Nature of Vibration in medium

Transverse Waves

When a transverse wave travels through a medium, the particles (or fields in the case of electromagnetic waves), in that medium oscillate at 90° (right angles) to the direction of motion of the wave.

Water waves and all electromagnetic waves are transverse waves.

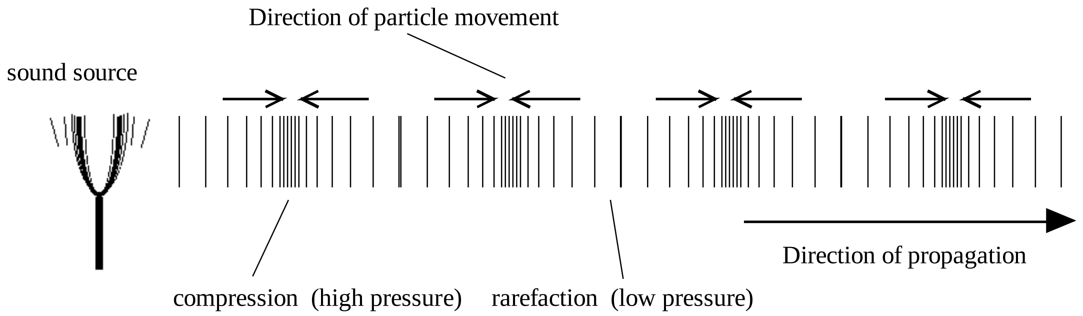


Transverse Wave

Longitudinal Waves

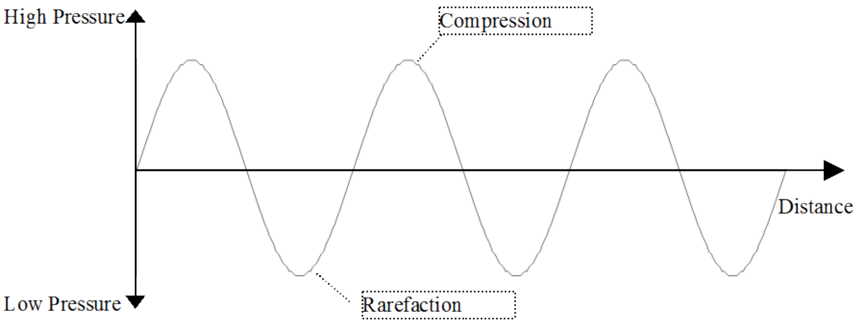
Particles in the medium through which a longitudinal wave is passing vibrate parallel to the direction of motion of the wave.

The movement of the particle results in a series of compressions and rarefactions which spread out from the source.



Longitudinal Wave

A graph of pressure against distance along a longitudinal wave produces a sine curve as shown.



Pressure Graph of a Longitudinal Wave.

Wave Characteristics

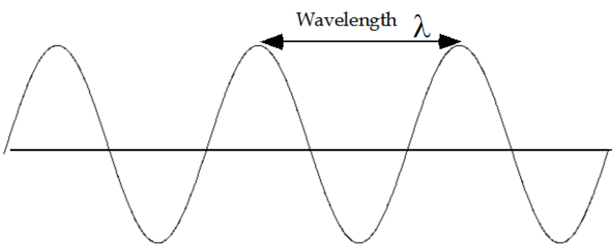
Wavelength

Wavelength (λ) is the distance between two successive points in phase.

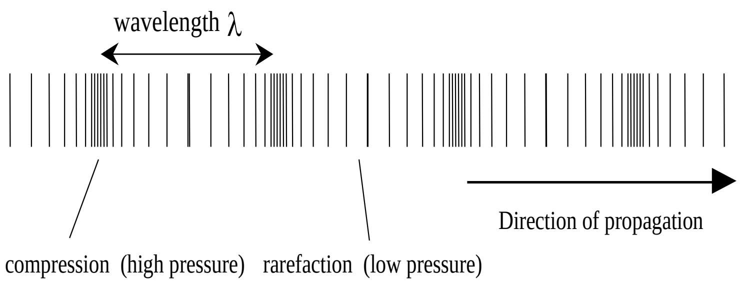
SI unit for wavelength is the metre (m).

For transverse waves this is the distance between two successive crests (or troughs)

For longitudinal waves this is the distance between two successive compressions or rarefactions.



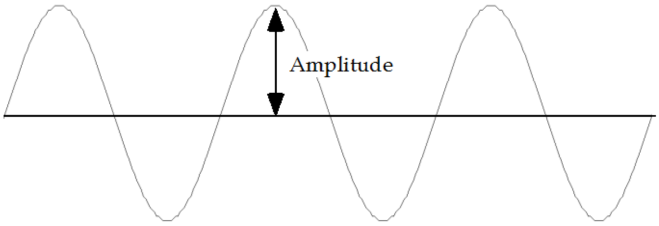
Transverse Wave



Longitudinal Wave

Amplitude

Amplitude (A) is the maximum displacement of a particle from its mean position.



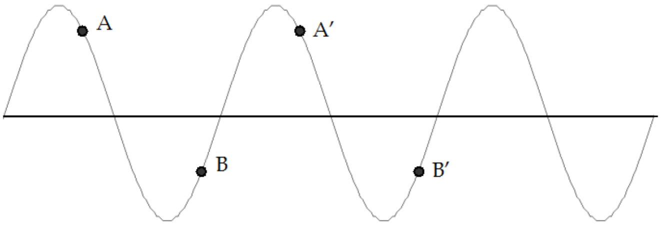
The amplitude is an indication of the energy content of the wave.

The greater the amplitude, the greater its energy content.

In sound waves an increase in amplitude is observed as an increase in the loudness of the sound.

Phase

Particles in a medium are in phase if they are undergoing identical motion (i.e. have the same velocity and displacement) at any given instant in time.



A and A' are in phase.

B and B' are in phase.

A and B are out of phase.

Angular Phase Difference

The simple harmonic motion of particles in a medium may be described in terms of their angular displacement.

One complete oscillation corresponds to an angular displacement of 2π radians (or 360^{o}).

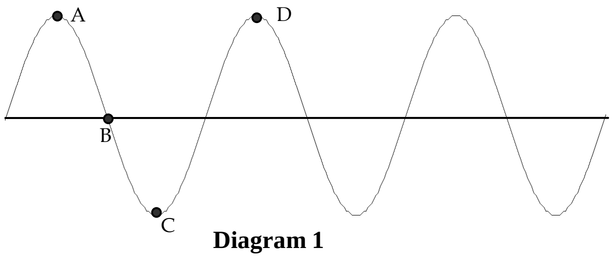


Diagram 1

The phase difference between two particles in a medium is expressed in terms of the difference in their angular displacements at a given instant.

In diagram 1, A is 90^{o}out of phase with B, and 180^{o} out of phase with C and in phase with D (i.e. angular phase difference between A and D = 0^{o})

The phase relationship of wave trains may also be described in terms of angular phase difference.

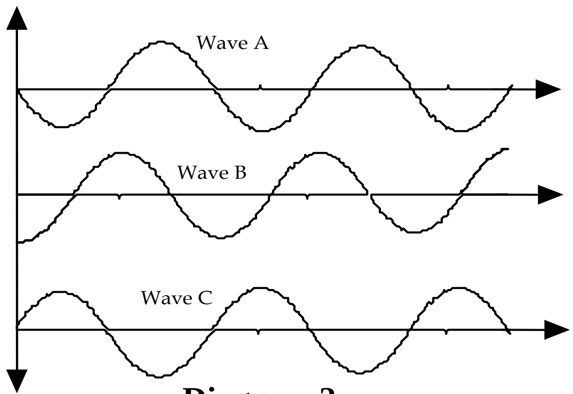


Diagram 2

In diagram 2, wave A is 90^{o} out phase with wave B and 180^{o} out of phase with wave C.

Period (T) is the time taken for a particle in the medium to complete one oscillation or vibration. It is also the time taken for one wavelength of a wave to pass a given point.

Frequency (f) is the number of oscillations or vibrations which occur per unit time. Frequency may also be defined as the number of wavelengths which pass a given point per unit time.

The S.I. unit for frequency is the hertz (Hz).

Frequency is the inverse of the period of a wave.

$$T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T}$$

Speed of Sound

The speed of sound in a medium is related to how quickly energy can be transferred between the particles in that medium. Overall this is found to depend upon how close the particles are to each other (density) and the forces that exist between them (elasticity).

As a general rule, there is an increase in the speed of sound in moving from gases to liquids to solids.

The speed of sound in air increases with increase in temperature.

Since the speed of particles in a gas increases with increase in temperature, the rate at which energy is transferred from particle to particle increases.

Pressure has negligible effect on the speed of sound in a gas.

The Wave Equation

As the speed of a wave depends upon the nature of the medium through which it is moving, the wavelength of the wave will depend upon the speed of propagation and the frequency of the source.

$$\text{Speed of a wave} = \frac{\text{the distance moved by the wave}}{\text{time taken}}$$

but the distance moved by the wave = (wavelength) x (number of wavelengths)

$$v = \frac{(\text{wavelength}) \times (\text{number of wavelengths})}{\text{time taken}} = \text{wavelength} \times \text{frequency}$$

Thus $\boxed{v = f\lambda}$ The speed of a wave is the product of its wavelength and frequency.

Problem:

An ultra sound signal is propagated through water with a frequency of 5.00×10^4 Hz.
Determine the wavelength of this signal given that the speed of sound in water is 1.50×10^3 ms^{-1}.

Notes:

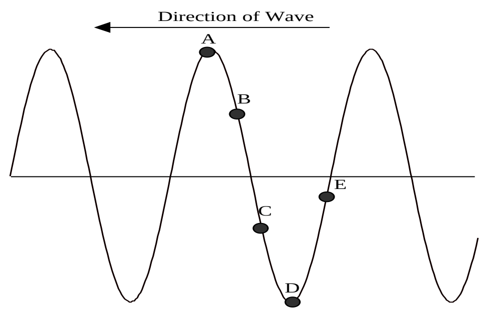
1. Do not confuse **ultrasonic** (high frequency i.e. f > 20 kHz) with **supersonic** (high speed i.e. speeds greater than the speed of sound in air (approx. 340 ms^{-1}).

2. Sounds with frequencies below the audible range (i.e. f < 20 Hz) are called **infrasonic.**

Exercise Set 1: Wave Characteristics

1. What is a wave?
2. Define the following terms a) frequency; b) wavelength; c) period; d) amplitude.
3. Sound will travel through air and water but will not travel through a vacuum. Explain.
4. Explain how longitudinal and transverse waves differ?
5. Classify the following waves as longitudinal or transverse:
   a) an ocean wave; b) an orange light wave; c) a 256 Hz sound wave;
   d) a wave travelling through a stretched string e) a Mexican wave.
6. Longitudinal waves consist of compressions and rarefactions.
   What are compressions and rarefactions?
7. Particle A oscillates in phase with particle B but 180° out of phase with particle C.
   Describe the motion of the particles.
8. Jason places his ear on an iron railway line. When the line is struck with a hammer, 1.00km away, Jason observes two separate sounds. a) Why does Jason hear two sounds?
   b) Find the time interval between the two sounds. (υ_{air} = 344 ms^{-1}, υ_{iron} = 5.20 x 10^3 ms^{-1})
9. Draw a diagram of a transverse wave. On the diagram indicate and label:
   i) the wavelength; ii) the amplitude; iii) the direction of propagation of the wave;
   iv) the direction of particle vibration; v) two points on the wave that are in phase.
10. Draw a diagram of a longitudinal wave. On the diagram indicate and label:
    i) the wavelength; ii) the amplitude; iii) the direction of propagation of the wave;
    iv) the direction of particle vibration; v) two points on the wave that are in phase.
11. The diagram illustrates a transverse wave moving through a medium in the direction shown at a particular instance in time.

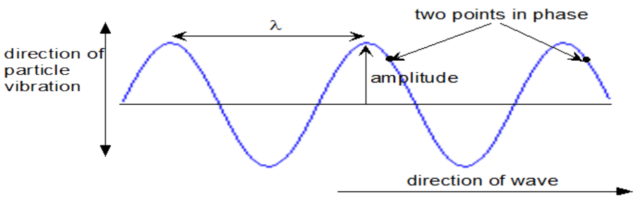
    Determine the direction the particles indicated are moving at this time.



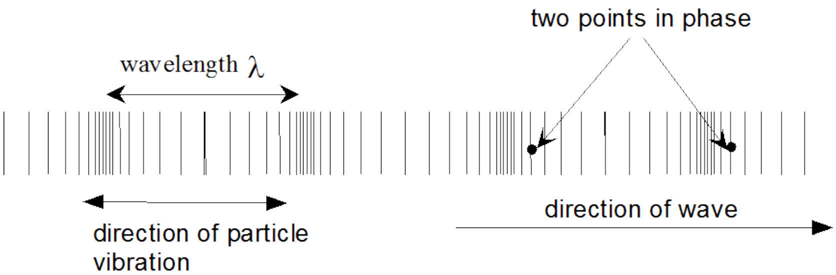
12. Determine the period of the following waves:
    a) ocean waves with a frequency of 0.0750 Hz;
    b) a low pitched sound of frequency 40.0 Hz;
    c) a high pitched sound of frequency 2.50 x 10^3 Hz;
    d) a visible light wave with a frequency of 5.00 x 10^{14} Hz.
13. Determine the frequency of a sound wave with a period of a) 3.90 ms b) 0.227 ms c) 20.0 μs
14. It is observed that the distance between successive crests in a ripple tank is 4.00 cm and that the dipper strikes the surface of the water 10.0 times in 2.00 s. For the wave determine:
    a) its wavelength; b) its period; c) its frequency; d) its speed.
15. The dipper in a ripple tank is observed to oscillate with a frequency of 20.0 Hz. If the water wave produced has a speed of 0.400 ms^{-1} determine: a) its period; b) its wavelength.
16. A boy throws a stone into a lake and observes that after 5.00 s 20.0 ripples have been produced. In that time, the outermost ripple has travelled 2.00 m from the point where the stone entered the water. Determine the waves: a) frequency; b) wavelength; c) speed

Answers

1. energy transfer without transfer of mass
2. a) number oscillations per unit time, b) distance between two successive points in phase, c) time for one complete oscillation d) maximum displacement from mean position
3. sound requires a physical medium (i.e. contains particles) a vacuum does not contain Particles.
4. particles vibrate a) same direction as wave b) at right angles to wave direction
5. trans, trans, long, trans, trans
6. a region where particles forced further apart (low pressure), a region where particles forced closer together (high pressure)
7. at any given time A and B move in the same direction, C moves in the opposite direction
8. a) Speed of sound in steel much higher than air thus difference between times taken for sound to travel through steel and air results in two sounds. b) 2.71s
9.



10.



11. stationary A,D moving down B,C moving up E
12. a) 13.3 s b) 25.0 ms c) 0.400 ms d) 2.00×10^{-15} s
13. a) 256 Hz b) 4.41 kHz c) 50.0 kHz
14. a) 0.0400 m b) 0.200 s c) 5.00 Hz d) 0.200 ms^{-1}
15. a) 0.0500 s b) 2.00 cm
16. a) 4.00 Hz b) 10.0 cm c) 0.400 ms^{-1}

Representation of Waves

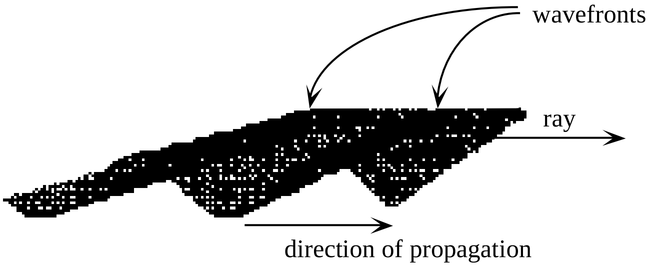
Waves may be represented 2 dimensionally as **wavefronts**.

A **wavefront** is a line representing a crest of a transverse wave or a compression of a longitudinal wave.

All points on a wavefront are in phase.

Wavefronts are separated by one wavelength.

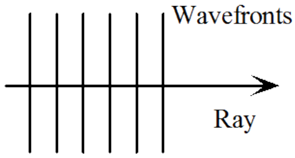
A **ray** is a line drawn at 90° to the wavefront and indicating its direction.



Plane waves consist of straight parallel wavefronts.

This type of wave is typical of waves that originate from a point source a long distance away.

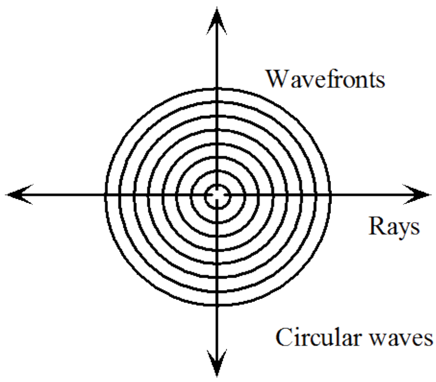
e.g. light from the sun may be considered to be a plane wave. A plane wave may also originate from a straight extended source that is close by. e.g. a water wave in a ripple tank.



Plane waves

Circular waves consist of concentric circular wavefronts.

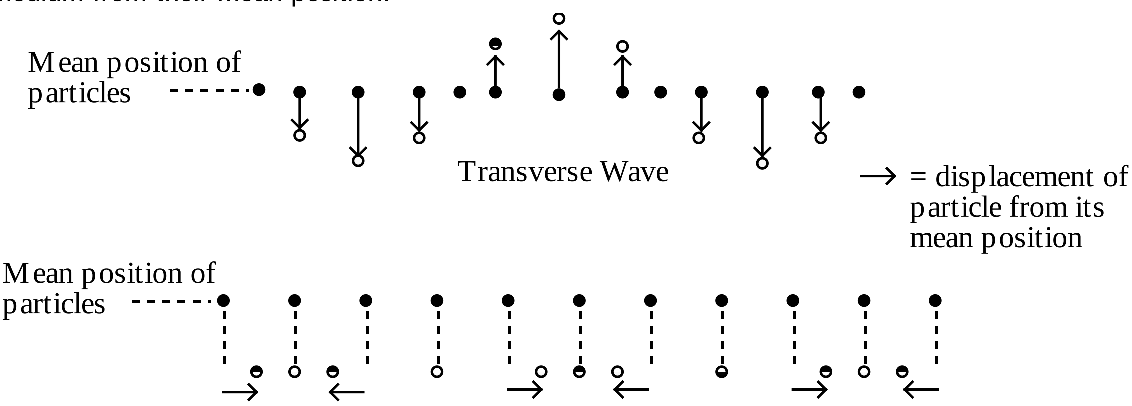
This type of wave is typical of waves which originate a short distance away from a point source.



Circular waves

Graphing Waves

Graphs are used to analyse wave motion. These graphs measure the displacement of particles in the medium from their mean position.



For both types of wave, each particle in the medium undergoes the same simple harmonic motion.

However, each particle's motion is slightly out of phase compared to the motion of the particles adjacent to it.

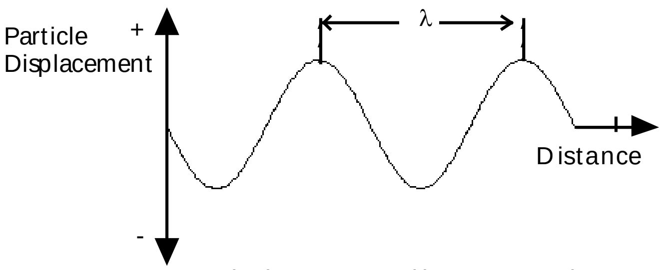
Graphing the motion of these particles for either type of wave will result in a sine curve.

Graphical Analysis of Waves

Graphs can be used to represent wave motion of both transverse and longitudinal waves.

1. Displacement - Distance

This is a "snapshot" of the displacement of the particles of a medium from their equilibrium position versus distance from the source of the disturbance at a particular instant in time. e.g.



Displacement – distance graph for a number of particles.

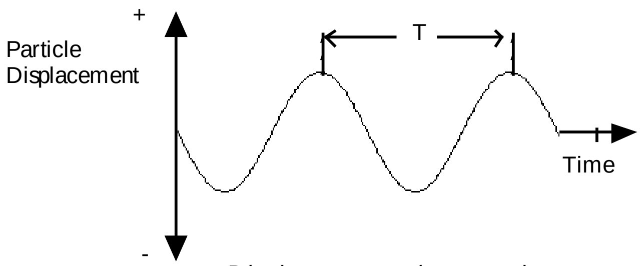
Note :

i) Since the particles may be displaced in either of two directions (up/down, left/ right) a sign convention must be used.

ii) The distance between crests represents the wavelength.

2. Displacement - Time

This type of graph shows the motion of a <u>single particle</u> of the medium over a period of time, as the wave passes. e.g.



Displacement – time graph for a single particle.

note :

The distance "T" on this graph represents the period of oscillation of the particle.

From this the frequency can also be determined.

TYPE EXAMPLE

A cork is placed in the pond 6.0 m from the source of the waves and the source is then made to vibrate. Graph the **displacement-time** graph of the cork's position over 5.5 s.

distance to cork = 6.0 m

time = $\frac{distance}{speed} = \frac{6.0}{4.0} = 1.5\,s$

speed = 4.0 ms^{-1}

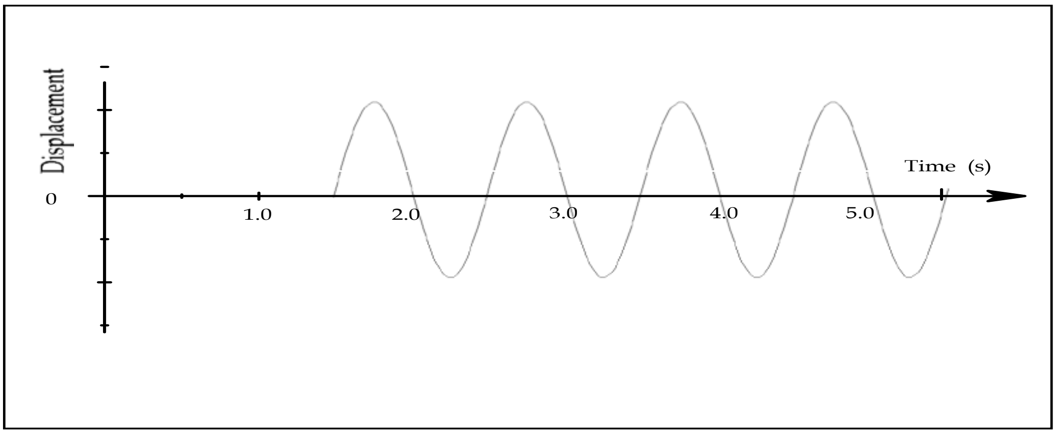
time to reach cork = ?

The wavefronts will not reach the cork for the first 1.5 s.

Also, as the frequency = 1.0 Hz, then the period = 1.0 s.

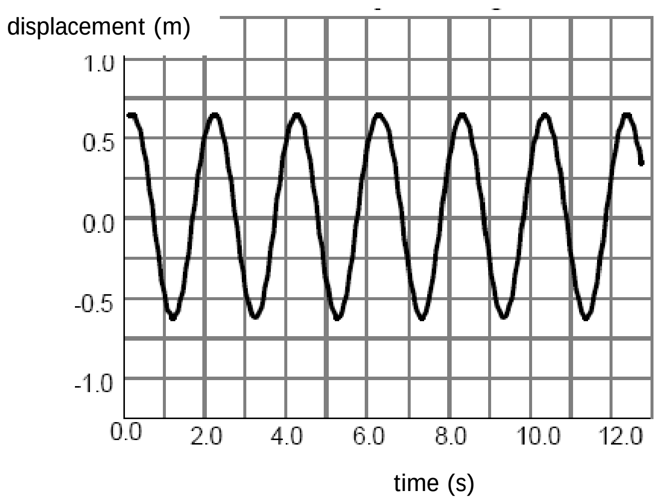
The cork will undergo 4 full oscillations in the first 5.5 s. (5.5 - 1.5)/1.

The first movement of the cork will be upwards as a crest reaches it first.



TYPE EXAMPLE

The following graph shows the displacement against time for an object undergoing wave motion. Determine the waves: (a) amplitude (b) period (c) wavelength, given v = 30 ms^{-1}

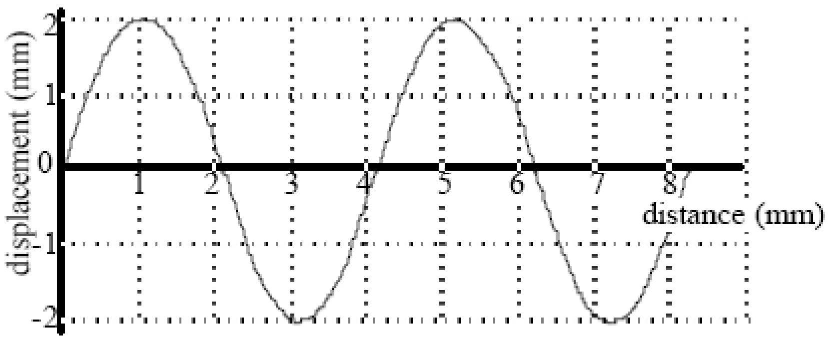


(a) 0.63 m
(b) 2.3 s
(c)

$$v = f \times \lambda$$
$$\Rightarrow \lambda = \frac{v}{f} \quad but \ T = \frac{1}{f}$$
$$\Rightarrow \lambda = v \times T$$
$$\Rightarrow \lambda = 30 \times 2.3$$
$$\Rightarrow \lambda = 69\, meter$$

TYPE EXAMPLE

The following diagram represents a waveform whose frequency is 50 Hz, at a certain instant in time.

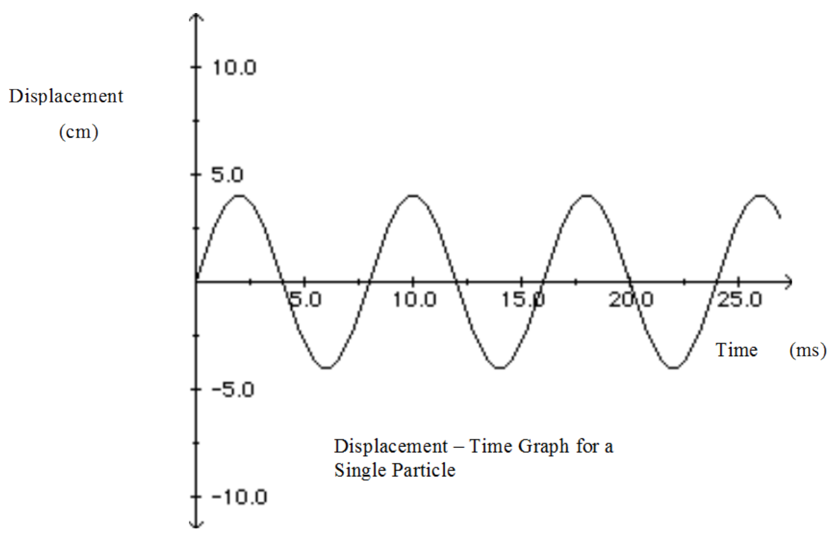


What is the wave's: (a) amplitude? (b) wavelength? (c) period? (d) velocity?

(a) 2 mm (b) 4.1 mm (c) $T = \frac{1}{f} = \frac{1}{50} = 0.020\,s$ (d) $v = f \times \lambda = 50 \times 0.0041 = 0.205\,ms^{-1}$

Problem 1: Consider the displacement time graph for a single particle in a medium.

Determine
a) the amplitude,
b) the period,
c) the frequency of the wave.



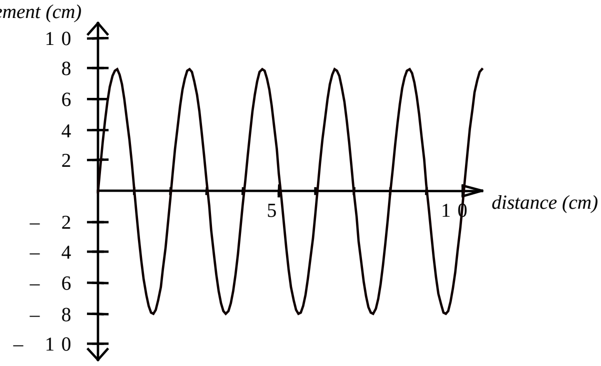
Exercise Set 2: Wave Graphs

Unless stated otherwise, the velocity of sound in air υ_{air} = 3.40 x 10^2 ms^{-1}.

1. How does a displacement - distance graph differ from a displacement - time graph?

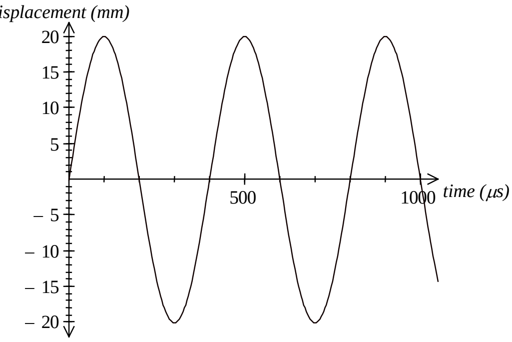
2. The graph illustrates the displacement of particles in a wave for a given time.

   Determine the wave's
   a) amplitude
   b) wavelength.



3. The graph illustrates the displacement of a particle, a given distance from the source of the wave.

   Determine the wave's
   a) amplitude
   b) period
   c) frequency.



4. An ocean wave has a wavelength of 20.0 m and is observed to move at 2.00 ms^{-1}.
   a) Draw a displacement – distance graph for a given time.
   b) Determine the frequency and period of the wave.
   c) Draw a displacement – time graph for a stationary boat as the wave passes beneath it.

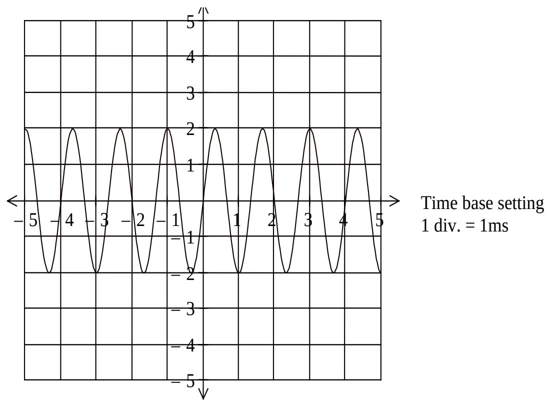
5. A sound from a musical instrument is known to have a frequency of 2.00 x 10^2 Hz.
   a) Draw a displacement-time graph to represent the wave.
   b) Draw graphs to represent the sound produced by the instrument with:
      i) the same loudness but lower pitch;
      ii) the same pitch but increased loudness.

6. The diagram illustrates the trace produced when a signal from an audio generator is fed into a cathode ray oscilloscope.

   The time base setting used is 1 div = 1ms.
   (i.e. Each division on the horizontal axis represents a time interval of 1ms.)

   a) Determine the period and frequency of the signal.

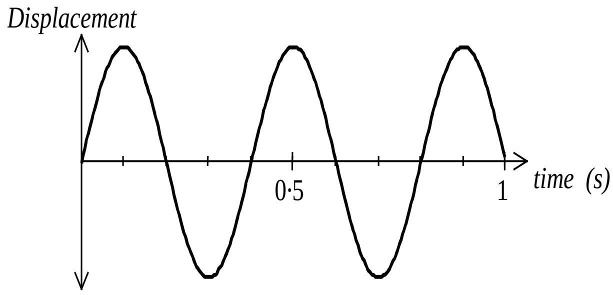
   b) The signal is amplified and fed into a loudspeaker.



   Draw a displacement – distance graph for the sound in air produced by the loudspeaker.

9. A dipper in a ripple tank starts oscillating at time t=0s. A displacement – time graph is provided for the source from the moment that it starts to oscillate.

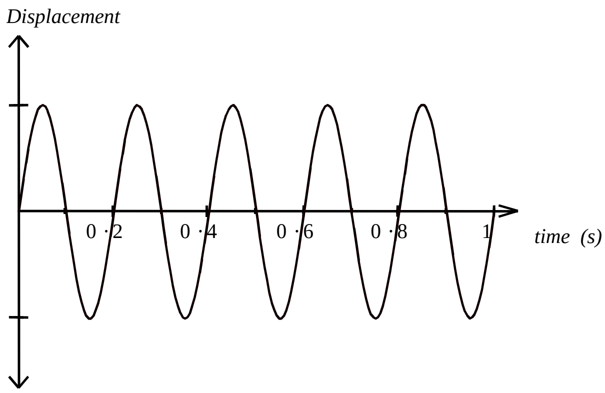
The wave produced in the tank travels at 15.0 cms^{-1}.



a) Draw a displacement – time graph (in the range t=0s to t=1.5s) for a point 7.5 cm from the source.

b) Draw a graph to show displacement against distance measured from the source at time t=0.80s.

10. A single source is used to drive two dippers in separate ripple tanks. The depth of the water in the two tanks is different and so the waves produced will travel at different speeds. A graph of displacement against time is shown for the source where t = 0 is the time the source starts to vibrate.

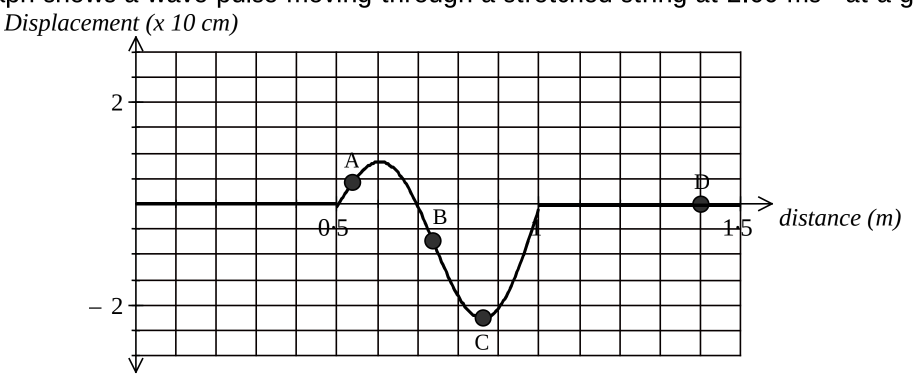


a) What is the frequency of the dippers?

b) Given that the velocities of the waves in the tanks are 0.100 ms^{-1} and 0.120 ms^{-1}, determine the wavelengths of the waves produced in each tank.

c) Draw separate displacement – distance graphs for the waves produced in both tanks at time t = 0.600 s.

11. The graph shows a wave pulse moving through a stretched string at 2.00 ms^{-1} at a given time.



a) Compare the speeds and direction of motion of particles at A, B, C and D
b) How long does it take the pulse to reach point D?
c) Draw a displacement – time graph for particle D taking time t = 0 s at the moment the above graph was drawn.

Answers

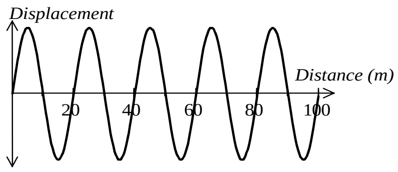
1. A displacement - distance graph must be drawn for a range of particles at a given time, while a displacement - time graph must be drawn for a single particle at a given distance from the source.

1. a) 15mm b) 0.04s c) 25 Hz

2. a) 8 cm b) 2 cm

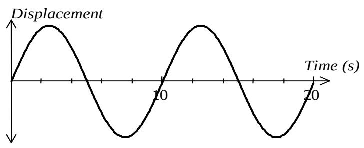
3. a) 20 mm b) 400 μs c) 2500 Hz

4a

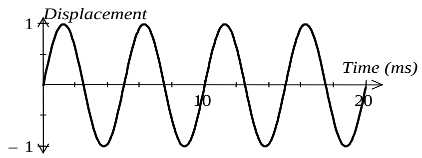


4b) 0.1 Hz, 10 s

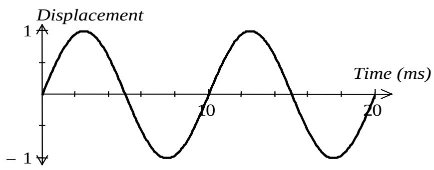
4c)



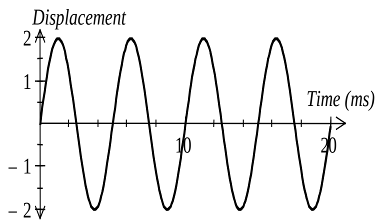
5a)



5bi)

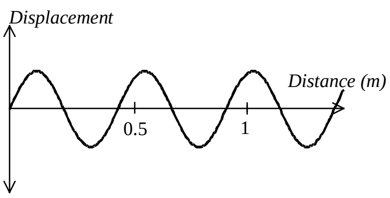


5b ii)

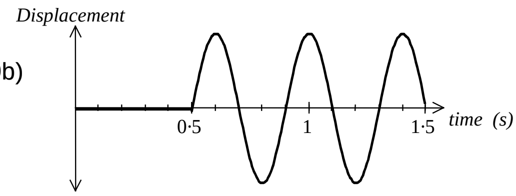


6a) 1.33ms, 750 Hz

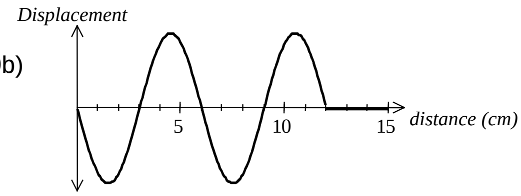
6b)



9b)

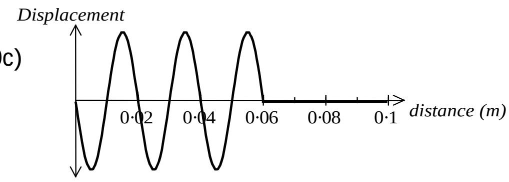


9b)

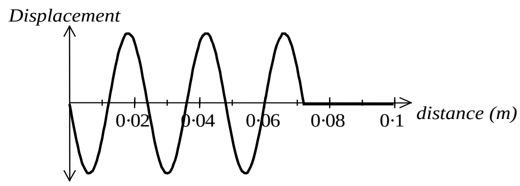


10 a) 5.0 Hz b) 0.02 m, 0.024 m

10c)



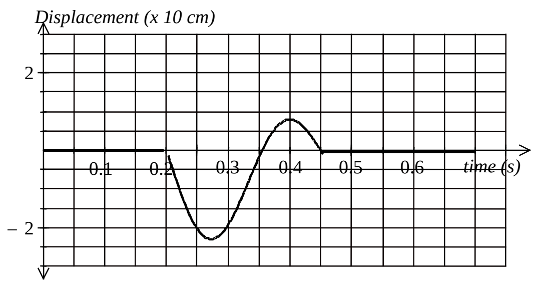
10c)



11 a) B up (≈ 4 ms^{-1}) faster than A down (≈ $2ms^{-1}$) C & D at rest

11 b) 0.20 s

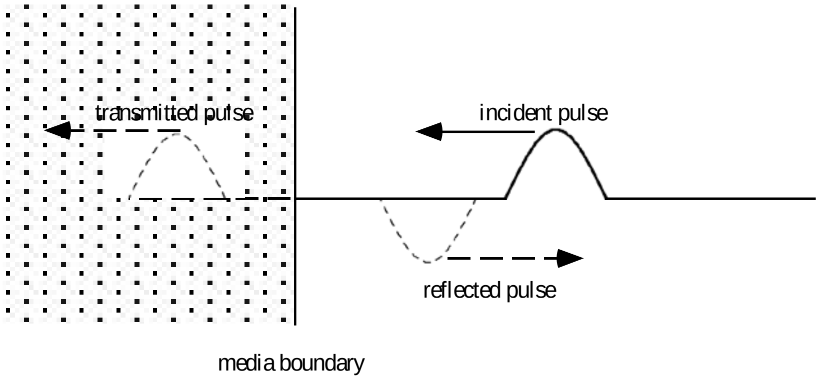
11c)



Wave Behaviour

Behaviour of Waves at Boundaries

When a wave is incident at a boundary between two media, some of the wave is transmitted while some of the wave is reflected.



The proportion of a sound wave that is reflected depends upon the relative difference between the acoustic characteristics of the two media.

The greater the difference in the impedance, the greater the amplitude of the reflected wave.

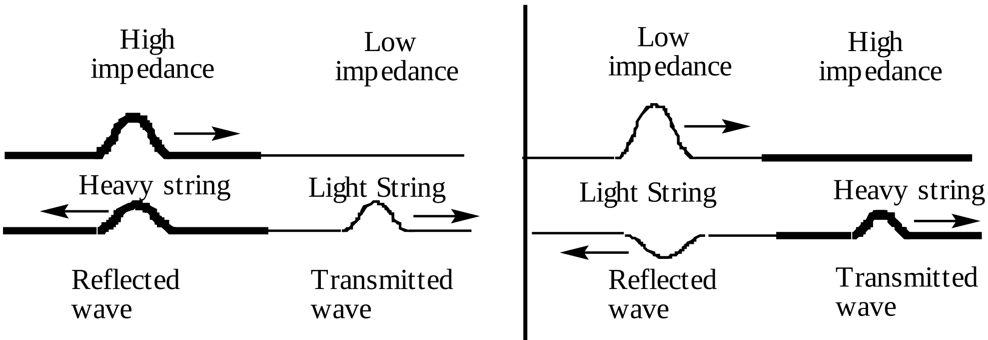
For example most of the sound is reflected when a sound wave in air strikes a brick wall (large difference in impedance).

However only a small amount of sound is reflected at the boundary between a region of hot air and a region of cold air (small difference in impedance).

Phase Changes at Boundaries

When waves are reflected from a medium of greater impedance the wave undergoes a 180^{o} phase change. When reflected from a medium of lower impedance there is no change in phase.

Examples: 1. Reflection and transmission in strings



(a) No phase change on reflection

(b) 180° phase change on reflection

In diagram (a), the pulse incident in a medium of higher impedance (thicker string) is reflected without a change in phase from the medium of lower impedance (thin string).

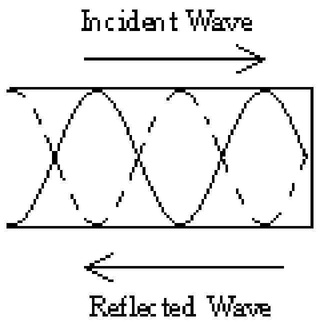
In diagram (b), the pulse incident in a medium of lower impedance (thin string) is reflected with a 180^{0} change in phase from the medium of higher impedance (thick string).

This example is similar to what happens with a guitar string - the wave travels up to the point of attachment on one side of the string and is reflected along the other.

Examples: 2. Reflection and transmission of sound in pipes

This can also be considered in terms of the impedance of the medium.

When a sound wave in a closed tube is reflected from its closed end, there is no change of phase when you consider the pressure wave.



Sound travels faster in the material making up the end of the pipe i.e. it has lower impedance than the air.

This example can also be thought in another way -as a compression strikes the end of the pipe, it becomes even more compressed against the unyielding surface and a compression is reflected.

The particle displacement in the medium is 90° out of phase with the pressure wave.

The pattern for particle displacement is as shown above.

Minimum particle displacement occurs when there is maximum variation in the pressure.

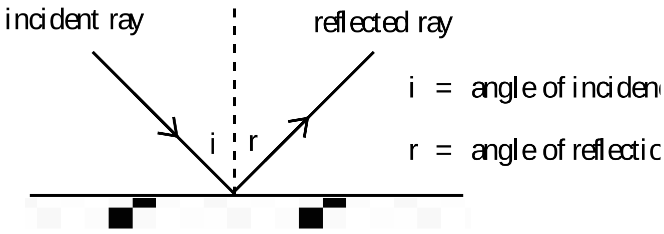
When a sound wave is reflected at the open end of a pipe, the pressure wave experiences a 180° phase change, as the air beyond the pipe has higher impedance. i.e. a compression is reflected as a rarefaction.

Reflection, Transmission and Absorption of Sound.

When sound is reflected it obeys the laws of reflection similar to light.

1. The angle of incidence is equal to the angle of reflection.

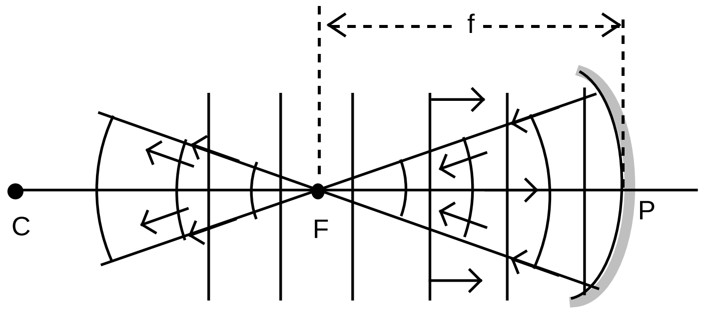
2. The incident ray, reflected ray and the normal all lie in the one plane.



Regular reflection occurs at flat surfaces and is responsible for echoes and reverberation.

When sound is reflected from spherical surfaces, it converges and diverges from concave and convex surfaces respectively.

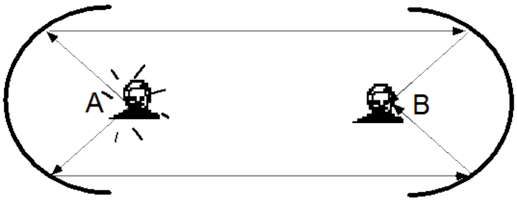
Sound reflected from a **concave** surface converges. Here sound waves converge towards F where the sound energy is concentrated.



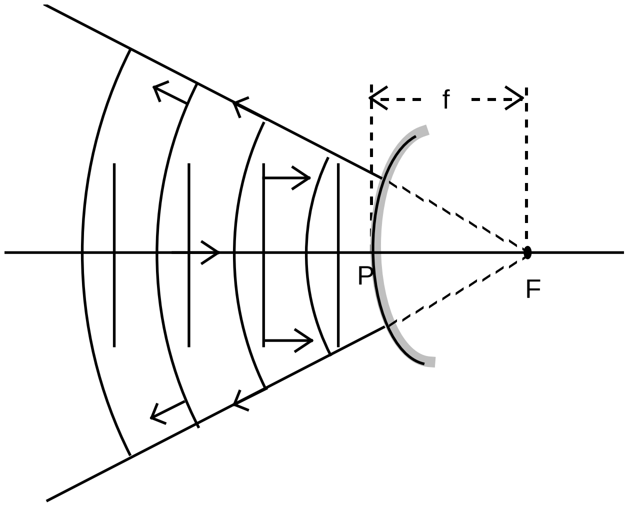
Concave reflectors can be used to transmit sound through air over relatively long distances with only marginal energy loss.

A whisper at A can be heard clearly by an observer at B even though they may be 50+ metres apart.

Note- A and B must be at the principal focus of each of the concave reflectors.



Sound reflected from a **convex** surface diverges.



Reflection occurs best at smooth, hard surfaces. Soft porous surfaces are poor reflectors.

Refraction

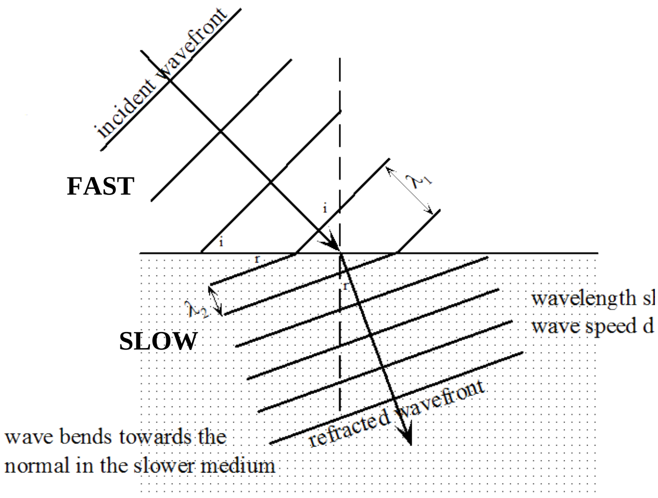
Refraction is the bending of a wave as it passes from one medium into another.

Refraction is due to a <u>change in the speed</u> of the wave as it passes <u>obliquely</u> into a new medium.

Waves are bent towards the normal when the wave's speed is reduced and away from the normal when the wave's speed increases.

The frequency of a wave does not change when it passes into a different medium.

The wavelength is directly proportional to the speed of the wave in the new medium.



As $f_1 = f_2$ and $f = \frac{v}{\lambda}$ $(v = f\lambda)$ □ then □ $\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$ or $\boxed{\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}}$

Thus the speed (or velocity) of the wave is directly proportional to its wavelength.

Snell's law states the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media.

This relationship was dealt with in year 11 for light. Here the constant was referred to as the relative refractive index of medium 2 compared to medium 1.

i.e. $$v_{2,1} = \frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

For sound this constant is equal to the ratio of the speed of sound in the respective media such that -

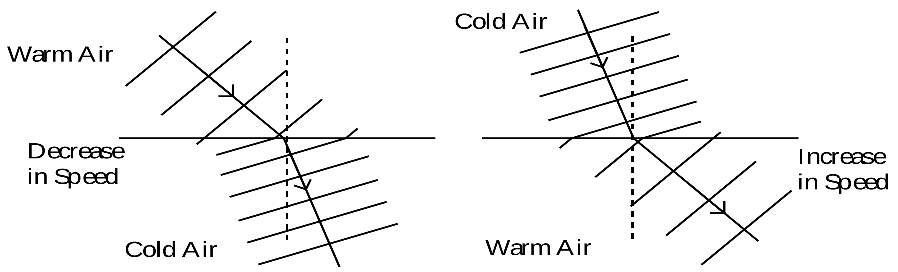
$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

Problem: A sound wave incident at 70.0° travels from a region of cool air at 10.0 °C to a region of warmer air at 20.0 °C. Determine the deviation of the sound wave given that the velocity of sound in air at 20.0 °C $v_{20\,C}$= 344 ms^{-1} and the speed of sound in air increases by 0.6 ms^{-1} for each 1 °C.

Atmospheric Refraction of Sound

The Effect of Temperature

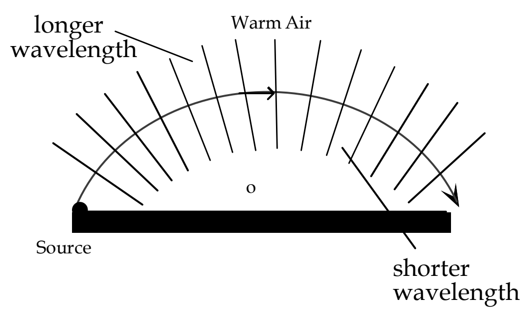
The speed of sound in air increases with temperature. Thus sound is refracted as it moves between bodies of air at different temperature.



On a still cool evening, sound waves may be refracted or totally internally reflected such that noise from a source many kilometres away can be heard clearly.

For this to occur, the air temperature must increase with height above the ground. A region where the temperature is increasing with height is called a **temperature inversion**.

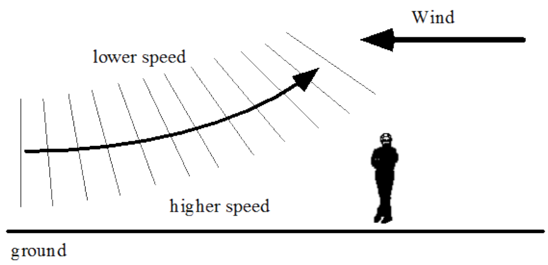
As the wavelengths of sound in the warm air gradually increase the sound wave will be refracted as shown in the diagram.



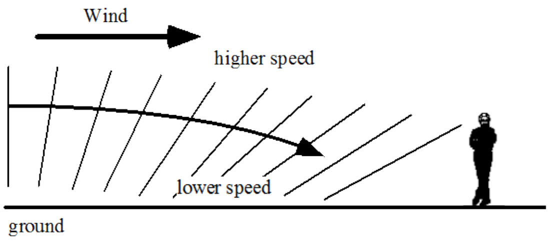
If the change is abrupt, total internal reflection may occur. All sounds greater than the critical angle will be reflected.

The speed of wind is normally faster above the ground compared with its speed close to the ground.

When sound travels in the opposite direction to the wind, the upper wavelengths are shorter than the lower wavelengths and the sound is refracted away from the ground and the observer.



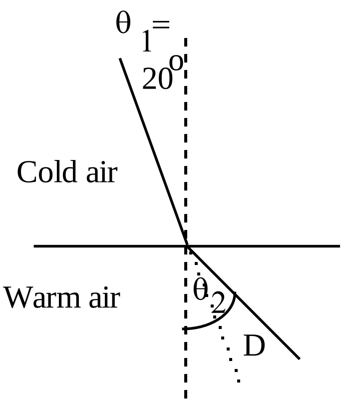
When sound travels in the same direction as the wind, the upper wavelengths are longer than the lower wavelengths and the sound is refracted towards the ground and the observer.



Why is it easier to hear a sound that travels with the wind than to hear a sound that travels against the wind?

TYPE EXAMPLE

A 800 Hz sound wave moves from a region of cold air where the wave travels at 320 ms^{-1} to a region of warm air where the wave travels at 340 ms^{-1}. If the wave has an incident angle of 20.0^0 , find the deviation of the wave and its wavelength in the cold air.



$$\frac{v_1}{v_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

$$\sin\theta_2 = \frac{\sin 20° \times 340}{320}$$

$$\theta_2 = 21.3°$$

D = 21.3 – 20.0 = **1.3°**

$$\lambda_2 = \frac{v_2}{f}$$

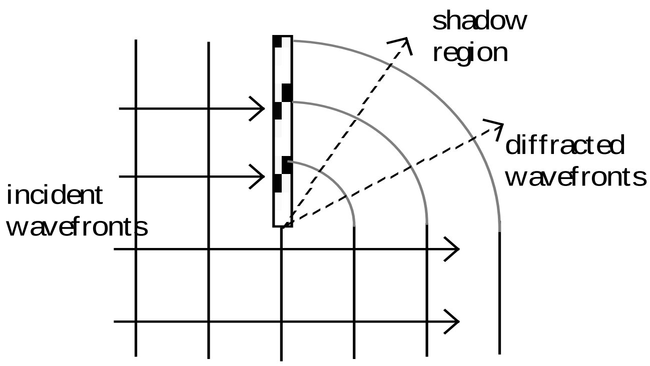
$$= \frac{320}{800}$$

= 0.400 m

Diffraction

Diffraction is the spreading of a wave about a barrier (or through an aperture).

In the diagram the wave was observed to have been diffracted into the shadow region behind the barrier.



Effect Of Wavelength On Diffraction

Diffraction increases with increase in wavelength. Thus long wavelength (low frequency) waves are diffracted more than short wavelength (high frequency) waves.

For example, when you pass through the Polly Farmer freeway tunnel, your radio can detect AM radio signals but not the FM band signals.

Ultrasound (high frequency sound), because of its short wavelength is not diffracted as much as audible sound.

Thus ultrasound (high frequency - short wavelength sound) is used in sonar and ultra sonic imaging to reduce dispersion of the beam due to diffraction.

Effect Of Aperture Size On Diffraction

Diffraction increases with decrease in aperture size.

In diagram 1, diffraction through the small aperture is more pronounced than diffraction through larger aperture shown in diagram 2.

The effect is maximised when the aperture is approximately the same size as the wavelength.

When the aperture is significantly smaller than the wavelength, the waves are blocked and energy does not pass through the aperture.

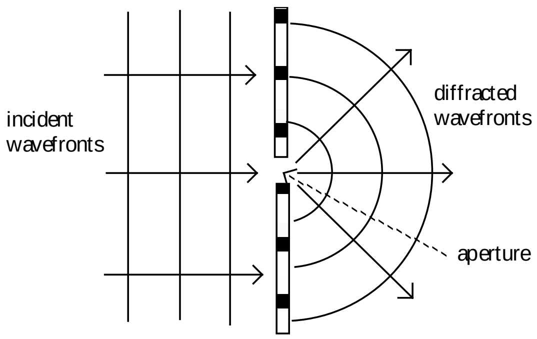
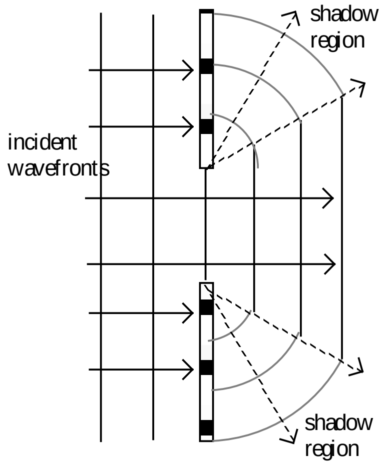


Diagram 1



Huygens' Principle

The phenomenon of diffraction may be explained in terms of Huygens' Principle which states that when a wave moves through a medium, the particles in that medium act as secondary sources.

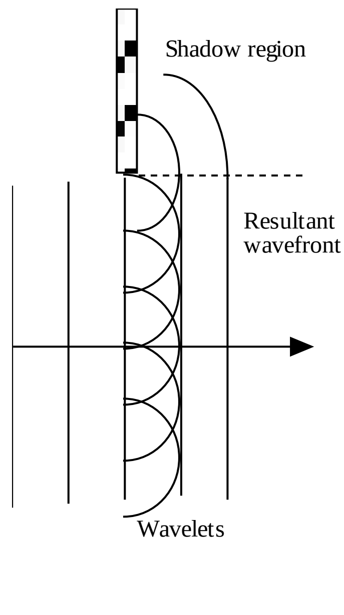
These particles oscillate with the same frequency generating circular or spherical wavefronts.

These wavefronts overlap and combine to form the resultant wave front.

In the diagram it can be seen that the individual circular wavefronts combine to produce plane wavefronts at positions away from the obstacle or barrier.

It also illustrates how circular waves are able to spread around the obstacle.

The intensity of the diffracted wavefronts decrease as fewer wavelets contribute to this part of the wave.



Diffraction of Sound

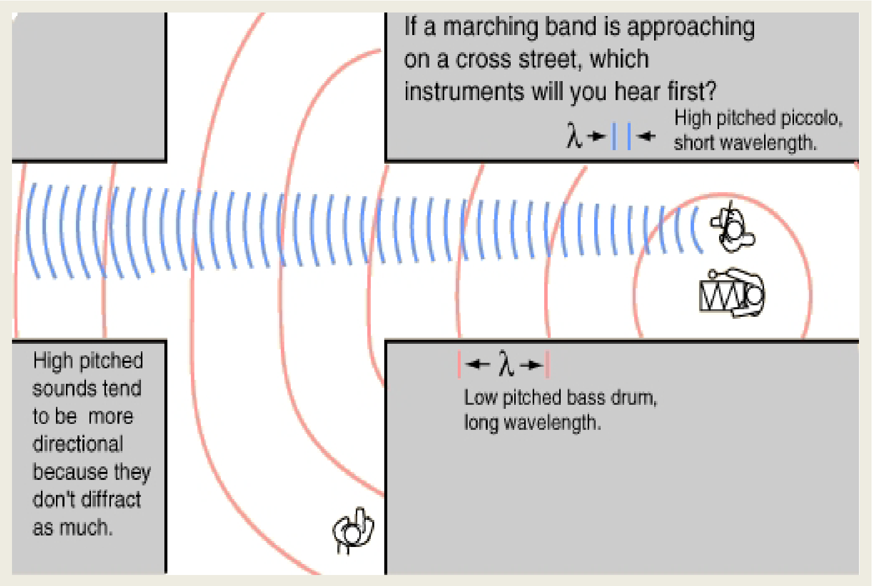
Audible sound has a relatively long wavelength (typically a metre or so) and thus is readily diffracted about obstacles and through apertures such as doorways.

Audible sound can be easily heard around corners for this reason, while visible light cannot.

To diffract visible light, apertures comparable to the wavelength of visible light must be used.

Ultrasound (high frequency sound), because of its short wavelength is not diffracted as much as audible sound.

Thus ultrasound is used in sonar to reduce dispersion of the beam due to diffraction.



Diffraction of Electromagnetic Radiation

The degree of diffraction of electromagnetic radiation also depends upon the wavelength of the respective bands of radiation.

Visible light, because of its small wavelength is not readily diffracted about every day obstacles and apertures.

Visible light can be diffracted if the aperture is small enough.

A fine line scored by a sharp razor blade, in a blackened plate, is small enough to produce diffraction of visible light.

However the amount of light that can pass through is so small that it is difficult to observe under normal conditions.

Relatively low frequency AM radio stations e.g. 1000 kHz can be detected over long distances as their long wavelength (≈300 m) allows for easy diffraction.

FM stations are on higher frequencies e.g. 96 MHz where the wavelength is shorter (≈ 3m).

FM radio stations (and TV sound which is sent on FM) tend to be "line of sight". .

Light Diffraction by a Razor Blade

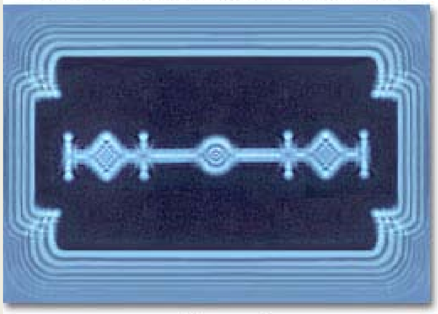


Figure 2

Exercise Set 3: Wave Behaviour

Unless otherwise stated or implied, speed of sound in air υ_{air} = 3.40 x 10^2 ms^{-1}.

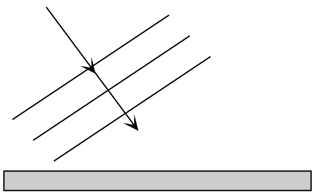
1. Explain what happens when a wave travelling along a stretched string is reflected from a) a fixed end b) a free end.

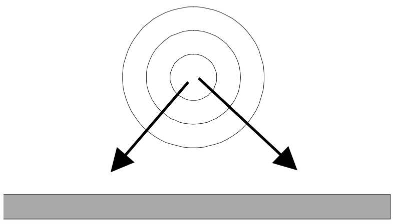
2. a) What is an echo? b) What are the conditions necessary for an echo to be heard?

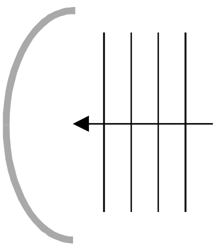
3. During an athletics carnival, a spectator hears two distinct sounds when a starting pistol is fired. The spectator observes that the starter is standing 50.0m in front of a plain brick wall when the pistol is fired.
   a) Explain why the spectator hears the two sounds.
   b) What is the time interval between the sounds?

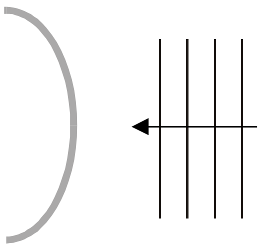
4. Many fishermen use echo sounders to locate schools of fish and to determine the depth of water beneath their vessels.
   a) Explain how echo sounders are able to indicated depth.
   b) Why do echo sounders use ultrasound rather than lower frequency sound waves.
   c) An ultrasonic pulse from an echo sounder is observed to return to a boat after 0.200 s. Find the sea depth beneath the sounder.
   [The speed of sound in water υ_{water} = 1.53 x 10^3 ms^{-1}.]
   d) A school of fish swim directly beneath the boat and result in a pulse returning to the boat in 0.150 s. How far above the sea floor are the fish swimming?

5. The following diagrams illustrate sound waves being reflected from rigid surfaces. Copy the following wave diagrams and complete them to show the reflected wave fronts.

a) 

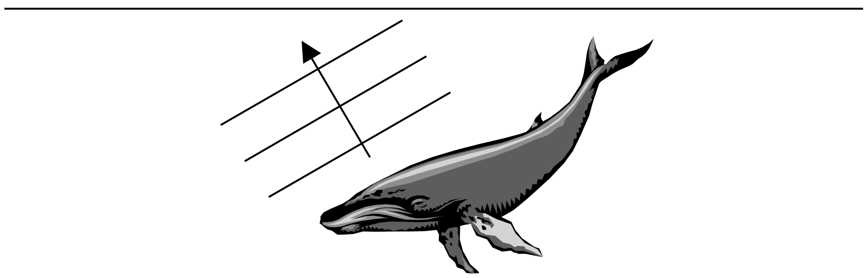
b) 


c)


d)

6. A sound wave with a frequency of 5.00 x 10^2 Hz, is incident at a boundary between two layers of air at an angle of 20.0°. The two layers of air are at temperatures of 20.0° C and 25.0° C and the sound wave is incident in the cooler medium. [Refer to page 7 of these notes for speed of sound information.]
   a) What is the frequency of the wave in the warmer medium?
   b) What is the period of the wave?
   c) What happens to the speed of the wave as it enters the warmer medium?
   d) Determine the wavelength of the sound in each medium.
   e) Through what angle is the sound wave deviated?

7. A whale produces a sound with a constant wavelength that is transmitted into the air above the surface.

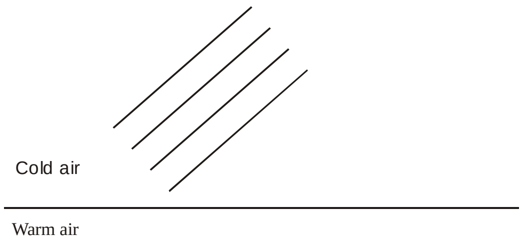


a) Complete the diagram to show the path taken by the wave as it passes from the water into the air.

b) If the wave is incident at 45.0° to the surface, determine the angle through which the wave is deviated.

8. The following diagrams show sound waves incident at a boundary between regions with different air temperatures. Complete the diagrams to show the path of the refracted wave fronts.

a)



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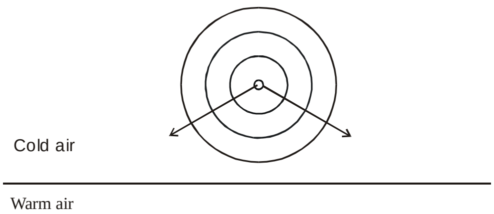


Answers

1. a) reflected wave inverted b) reflected wave not inverted
2. a) a reflected sound heard that can be distinguished from the original transmitted sound
   b) time between transmitted and reflected sound is greater than 0.1 s i.e. minimum path difference ≈ 35 m, reflector having a large surface area relative to wavelength.
3. a) sound reflected from wall produces an echo b) 0.294 s
4. a) time between transmitting the sound and receiving the echo enables depth to be determined
   b) shorter wavelength gives less diffraction thus more efficient
   c) 153 m 4d) 38.3 m (Hint: the sound has to travel down and then back up again.)

6 a) 500 Hz b) 0.002 s c) bends away from the normal d) 0.686 m, 0.692m e)0.18°

7 b) 36.0°

Wave Interactions - SUPERPOSITION AND INTERFERENCE

Principle of Superposition

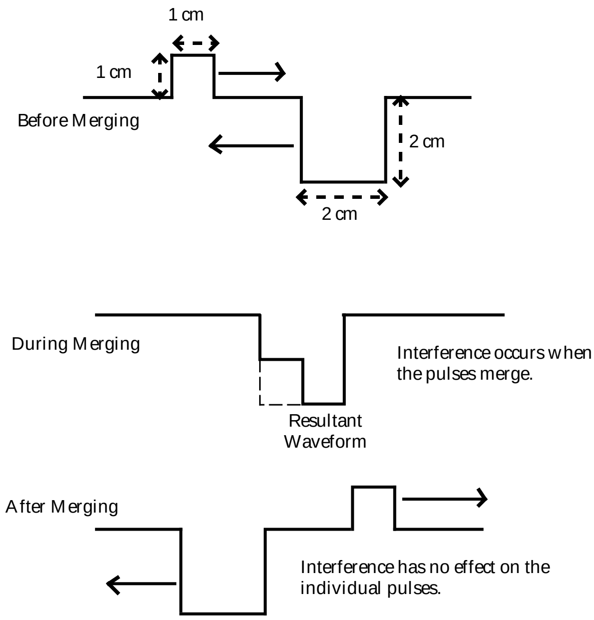
When two or more waves meet travelling in the same medium, the resultant displacement of the particles in the medium is the algebraic sum of the displacements due to the separate waves.

Consider two approaching pulses in a given medium.

When the pulses merge, the resultant shape is found by algebraically adding the displacements of the two waves together.

The resultant waveform produces an interference effect created by the merging of the two waves.

After the pulses have passed through each other they continue unaffected.



The interaction of waves undergoing superposition is termed **interference**.

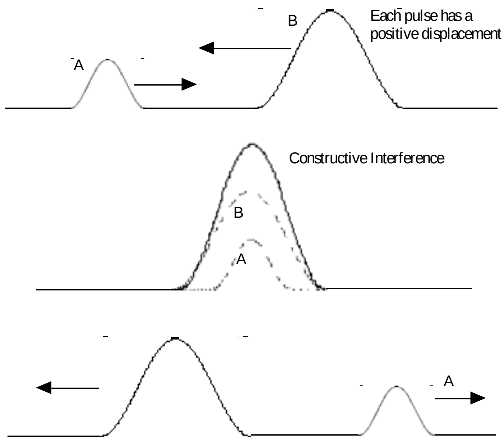
Constructive Interference

Constructive interference occurs when waves meet and have displacements in the same direction.

The displacement of the resultant waveform is greater than the displacements of either of the component waves.

Constructive interference results in a net increase in the amplitude and thus the energy content of the wave increases.

When sound waves constructively interfere there is an observed increase in the loudness.



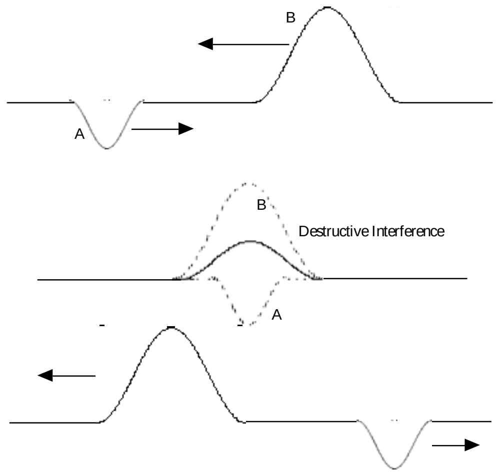
Destructive Interference

Destructive interference occurs when waves with opposite displacements merge.

The resultant displacement is less than the displacement of either of the component waves.

Destructive interference is characterised by a reduction in the amplitude.

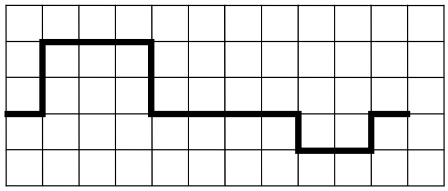
The energy content of the wave decreases.
When sound waves destructively interfere
There is a reduction in the loudness.



Exercise Set 4: Interference and Superposition

1. The two wave pulses approach each other with a combined speed of 2.0 cms^{-1} .

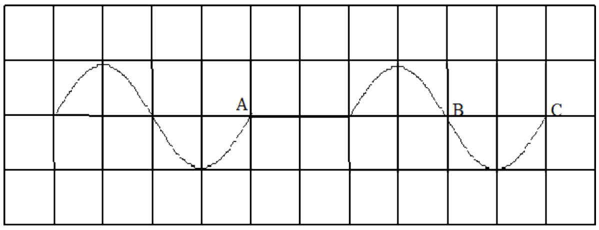
   Draw on graph paper the waveform that results after a) 2 s, b) 3 s, c) 4 s, d) 5 s



1 square = 1 cm

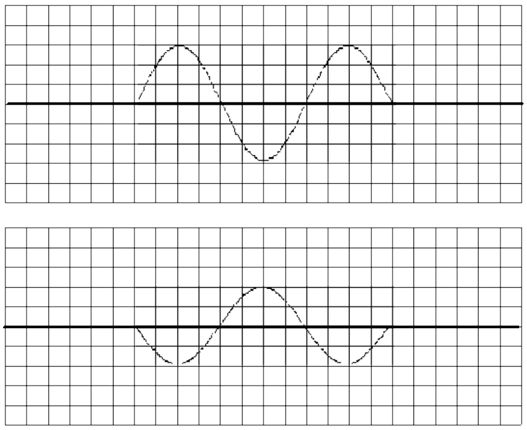
2. Two identical wave pulses approach each other.

   Sketch the waveforms which result when
   a) point A coincides with point B, and
   b) when point A coincides with point C.

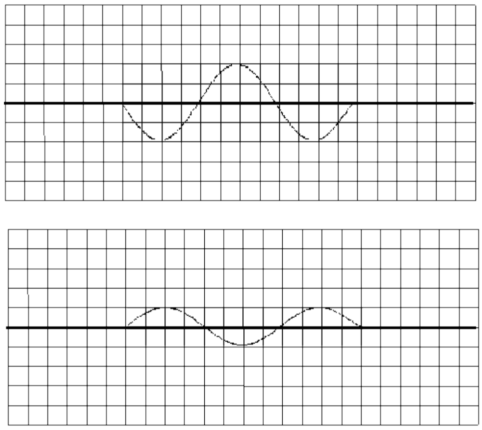


<u>For Q 3 - 6, draw the resultant wave if the two waves were superposed.</u>

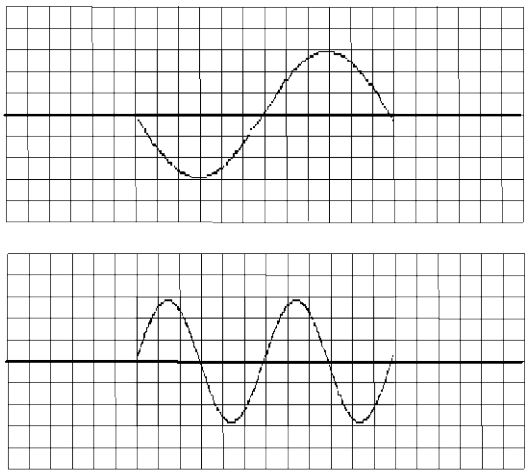
3.



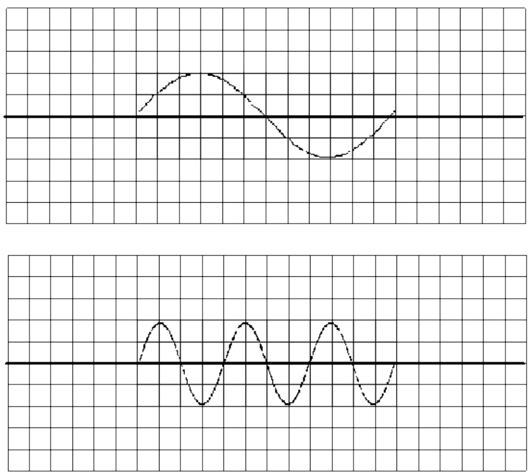
4.



5.



6.



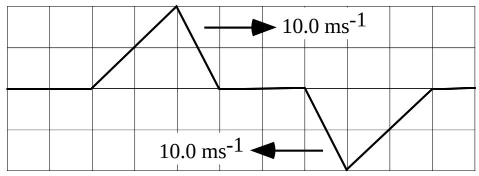
7. Describe what you would expect to see when two identical waves travelling in almost the same direction merge together with the crests of one wave coinciding with the crests of the other.

   What would be observed if the crests of one wave coincide with the troughs of the other?

8. Two wave pulses travel at 10 ms^{-1} as shown.

Draw the string 0.20 s later.

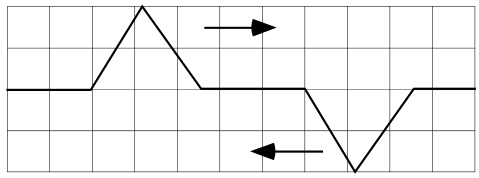
1 square = 1 m



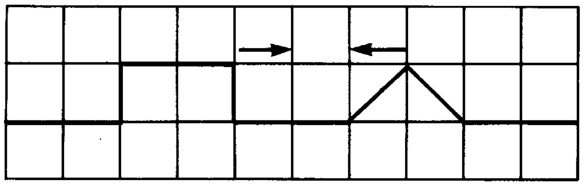
9. Consider the two pulses travelling toward each other along a string as shown.

How will the string appear at the instant the centres of the pulses coincide?

1 square = 1 m

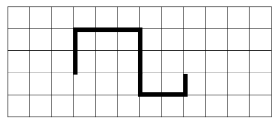


10. Redraw the two pulses depicted.
Use the principle of superposition to determine the shape of the wave when the centres of the pulses coincide.

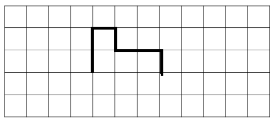


Answers

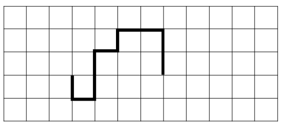
1 a)



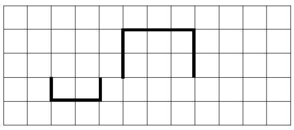
b)



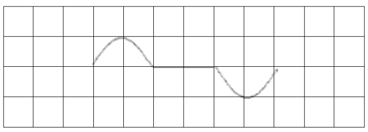
c)



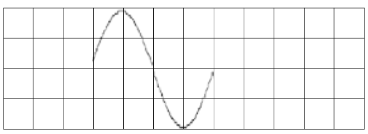
d)



2a)



2b)



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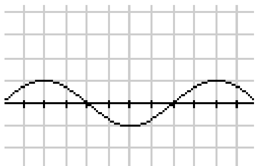

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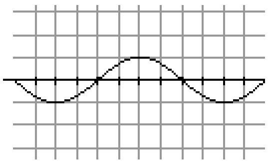

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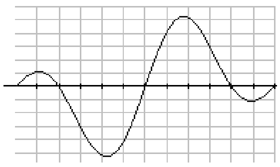

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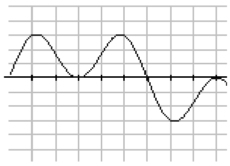
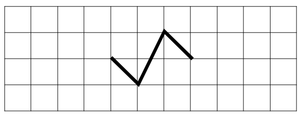


7 a) waves will reinforce each other, resultant wave will have double the amplitude,
b) waves will annul each other, resultant zero amplitude

8



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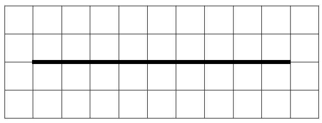

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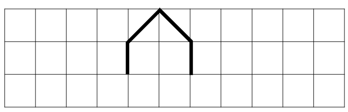
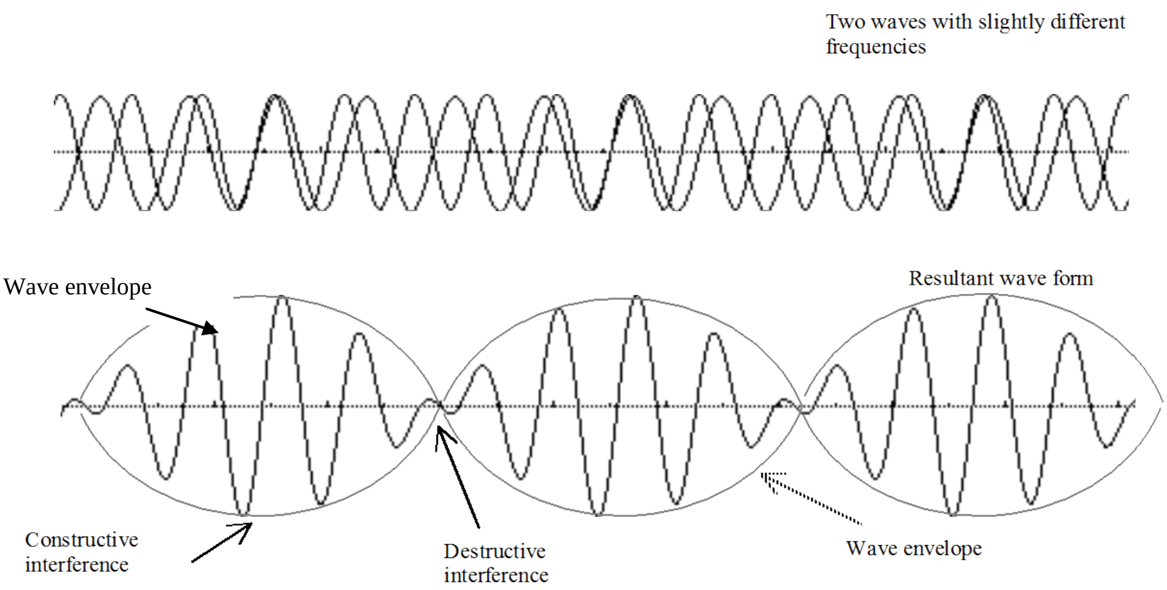


Beats

When waves which have slightly different frequencies superpose, the resultant interference gives rise to periodic variations in amplitude or loudness.

The periodic rises and falls in loudness are termed beats. When they are in phase they constructively interfere and when they are out of phase they destructively interfere.



The number of beats that occur each second is referred to as the **'beat frequency'**.

The beat frequency f_b is found to be the difference between the frequencies of the component waves.

$$f_b = |f_1 - f_2|$$

Beats may be observed when two motor cars are standing together at traffic lights. The exhaust notes from each car may have slightly different frequencies. The interference of these sounds results in a periodic rise and fall in the loudness of the combined sound.

A tuning fork can be used to tune musical instruments or help in the determination of an unknown frequency.

When the tuning fork is sounded together with a sound of unknown frequency the number of beats heard will be equal to the difference between the two frequencies.

The addition of a small piece of wax or plasticine to one of the forks of a tuning fork lowers its frequency and can help to determine which of the two possible frequencies is correct.

TYPE EXAMPLE

A tuning fork sounds at 256 Hz with a guitar string of unknown frequency. If 4.0 beats per second are heard, determine the possible frequencies of the guitar string.

f_b = 4 Hz
f_1 = 256 Hz

$$f_b = |f_1 - f_2|$$

$$f_2 = f_1 + f_b \text{ or } f_1 - f_b$$

$$= 256 + 4 \text{ or } 256 - 4$$

$$= 260 \text{ Hz or } 252 \text{ Hz}$$

TYPE EXAMPLE

Assume a small piece of wax is added to the tuning fork in the example above and reduces the beat frequency to 2.0 beats per second. Determine which of the frequencies for the unknown is correct.

The addition of wax lowers the frequency of the fork.

Since the number of beats decreases then the unknown frequency must be less than the original frequency of the fork.

So, **unknown frequency** (< 256 Hz) **= 252 Hz**

Problem:

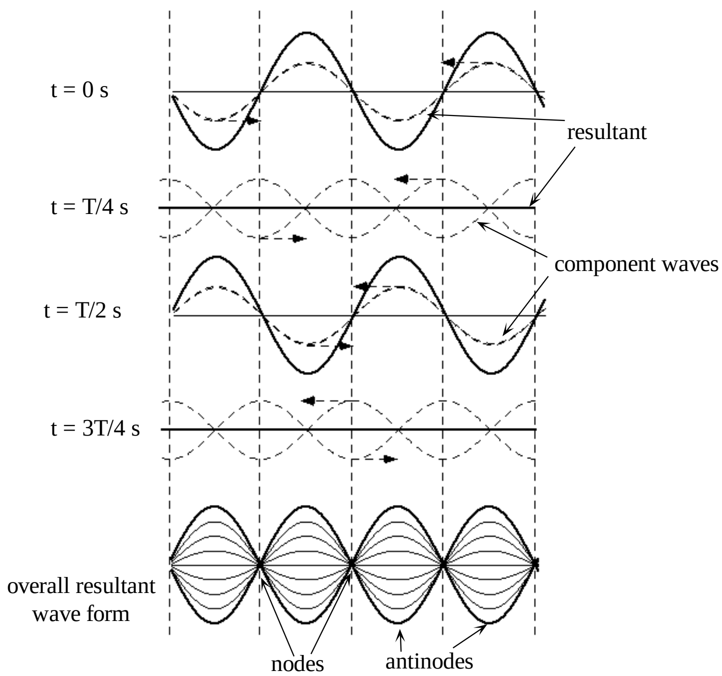
Find the frequency of an unknown tuning fork if it produces 4 beats when sounded together with a piano string of frequency 200.0 Hz, given that a small piece of wax loaded onto the tuning fork is observed to lower the number of beats. [The piece of wax will lower the frequency of the tuning fork.]

Standing Waves

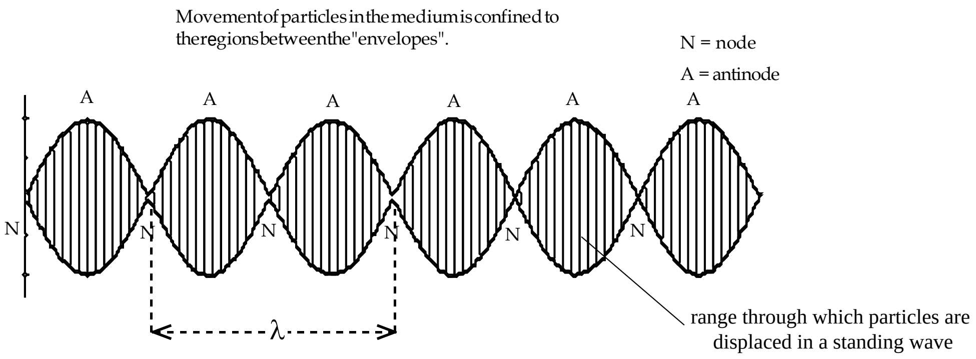
Standing waves (cf. progressive waves) are formed by the interference of two **coherent waves** travelling in **opposite directions.**

Consider two coherent waves travelling in opposite directions.

Note that the **SOLID LINE** indicates the resultant waveform.



Observed Standing Wave Pattern



Observed Standing Wave Pattern

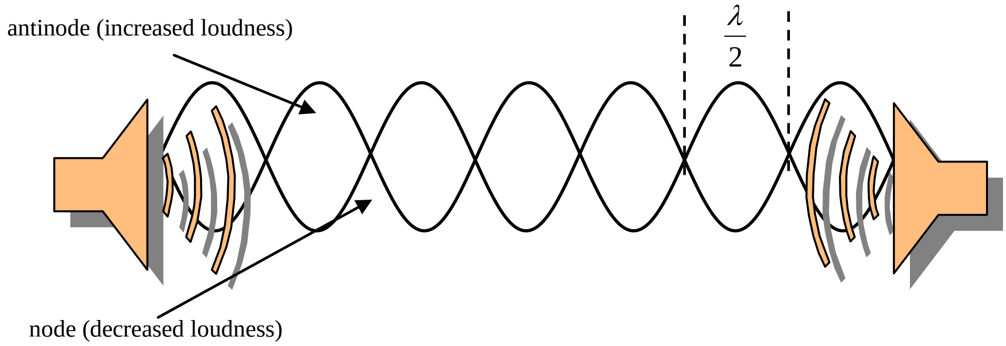
Characteristics of a Standing Wave

- The resulting standing wave does not progress (move) in either direction.
- Standing Waves are an interference pattern.
- Positions occur where there is no movement. These positions are termed *nodes*.
- Particle amplitude increases away from the nodes and is at a maximum at positions mid way between them. These positions are termed *antinodes*.
- The distance between successive nodes (or successive antinodes) is $\frac{\lambda}{2}$
- Movement of the particles in the medium and the energy of the wave is confined to the regions between successive nodes.
- The phase of all particles between adjacent nodes is the same. Particles in adjacent segments are 180^0 out of phase.

The diagram below shows two loud speakers being fed the same signal. That is, they are coherent sources. Note the points of increased loudness (i.e. Antinodes) and decreased loudness (i.e. Nodes).

This interference effect is not observed when a stereo signal is fed to a pair of speakers since the 2 signals are different and thus not coherent.

The distance between successive nodes or antinodes is $\frac{\lambda}{2}$.



Problem:
Two loudspeakers face each other and are separated by a distance of 1.20 m. They are connected to the same output from an audio generator that produces a 1.00×10^3 Hz signal.

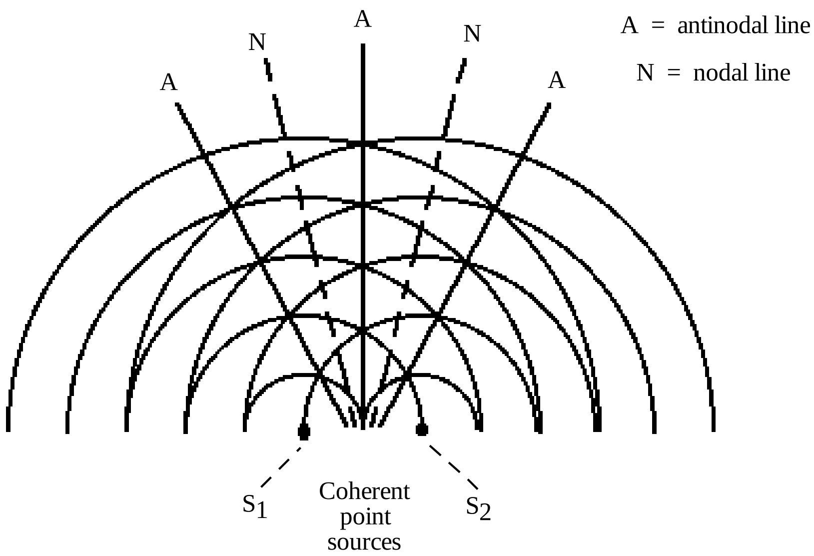
a) Why, in the above example, does an antinode have to occur mid-way between the speakers?
b) How many nodal positions (positions where there is a decrease in sound intensity) will occur between the two speakers?

Interference from Two Coherent Point Sources

Consider two oscillating coherent point sources generating circular waves.

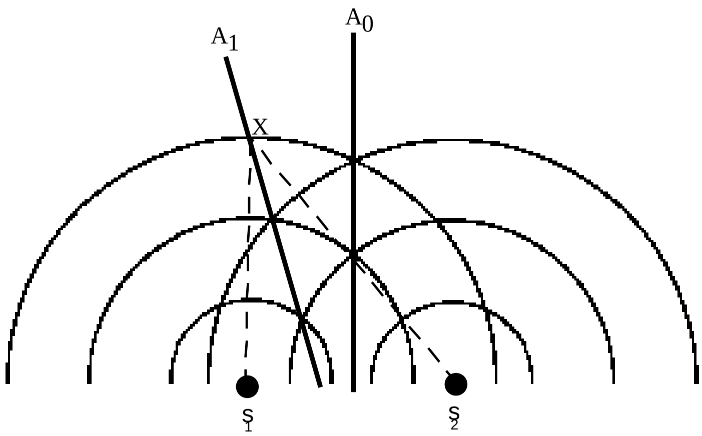
The superposition of the circular wave fronts results in the formation of lines along which constructive interference occurs. These lines (A) are termed antinodal lines.

Mid way between these lines destructive interference occurs resulting in the formation of nodal lines.



Path Difference

Path difference is the difference in the distances to a given point from each source producing the interfering waves (i.e. S_1 and S_2).



In the diagram the path difference (to point X from sources S_1 and S_2) PD = $S_2X - S_1X$.

Notice, for any point on the antinodal line A_1 the path difference, PD = λ thus the interfering waves are always in phase at points on this line.

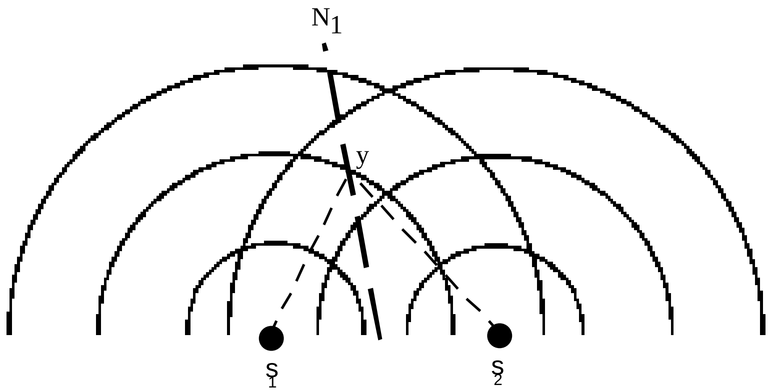
For all points on the antinodal line A_0 the path difference PD = 0, thus the interfering waves are also in phase for all points on this line.

Line A_1 is also an antinodal line. All points on the line A_1 have a Path Difference of exactly one wavelength.

Thus on any two antinodal lines, all points will have a path difference from S_1 and S_2 that must be a whole number multiple of the wavelength (i.e. $n\lambda$ where $n = 0, 1, 2, 3, \cdots$)

For any point on a nodal line, the interfering waves are 180^{o} out of phase and thus annulment occurs.

In the diagram the path difference to a point on the first nodal line (N_1) is $PD_{N1} = \frac{1}{2} \lambda$



For any given point on a nodal line (N) the waves are 180^0 out of phase. Thus the path difference is $(n - \frac{1}{2})\lambda$ where n = 1,2,3...

TYPE EXAMPLE
Two loudspeakers are separated and supplied with the same signal from an audio generator.

A physics student walks along a line XY as shown.

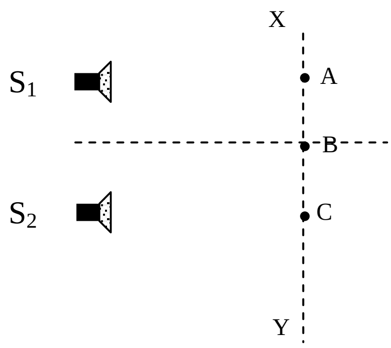
He notices the sound reaches a maximum at point A and then a minimum at point B.

The distance S_1A is 5.00 m and the distance S_2A is 6.50 m.

a) What is the wavelength of the sound ?

b) What is the path difference of the two sets of waves at B ?

c) What is the path difference of the nodal point observed by the student before reaching A?



a) at A

$PD = \lambda = 6.50 - 5.00$

i.e. $\lambda = 1.50$ m

b) at B

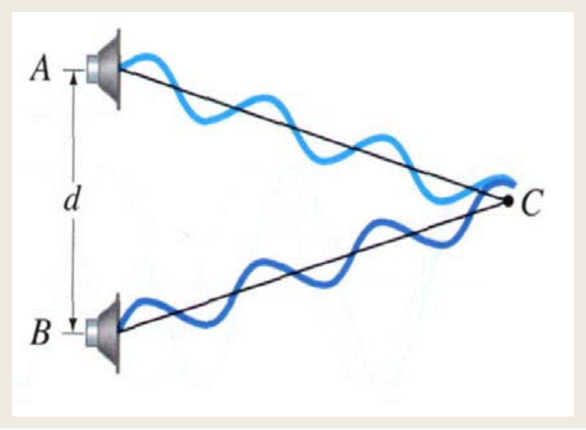
$PD = \frac{\lambda}{2} = \frac{1.50}{2}$

$= 0.75$ m

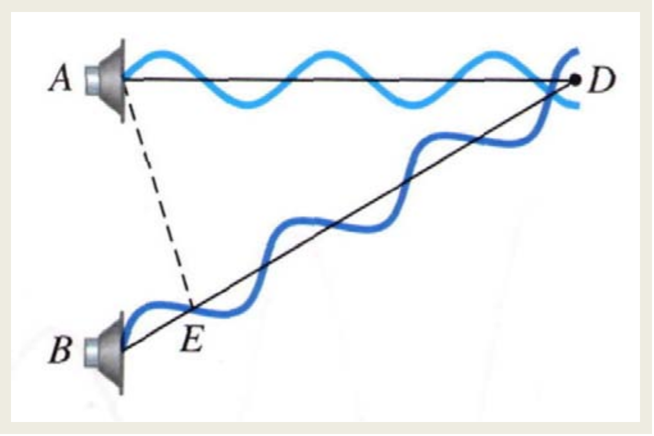
c) At the nodal point before A

$PD = \frac{3\lambda}{2} = 3 \times 0.75$

$= 2.25$ m



Constructive Interference



Destructive Interference

Exercise Set 5: Interference and Superposition

1. The phenomenon of beats can be used to tune a piano. Explain how this is possible.
2. What is the beat frequency heard when a 320 Hz sound and 330 Hz sound are played simultaneously?
3. Two loudspeakers are connected to the same signal generator in order to produce sound which has frequency 3.40×10^2 Hz. If the speakers are separated by 2.00 m, how many nodal positions will exist between the speakers? (Take the speed of sound as 3.40×10^2 ms^{-1} .)
4. For any given point on an antinodal line, what is the difference in the distances to each of the sources?
5. If there is a loud sound at a point where the paths difference between two coherent sound sources is 0λ determine the type of sound at points where the path difference is:

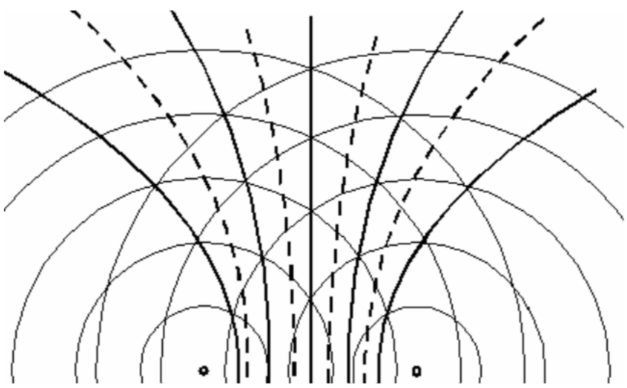
   a) 0.5λ b) 1λ c) 1.5λ, d) 2.0λ □

6. Two loud speakers separated by 1.0 m emit sounds of frequency 1.36×10^3 Hz. Assume that the speed of sound in air is 3.40×10^2 ms^{-1} and the speakers are in phase.

   a) Draw to scale the interference pattern that is produced.
   b) If the frequency of the sounds were doubled explain the change in the pattern.
   c) If the original frequency was used but the speakers moved 2.0 m apart, what effect would this have?

Answers

1. tune until the beats disappear
2. 10 Hz
3. 4
4. must be an integral (whole) number of wavelengths i.e. $n\lambda$
5. a) soft b) loud c) soft d) loud.
6. a)



6 b) distance between wavefronts decreases and distance between nodal and antinodal lines decreases

6 c) distance between wavefronts remain the same and distance between nodal and antinodal lines decreases

FORCED VIBRATIONS

When a swing is displaced sideways and released it begins to oscillate freely.

Unless supplied with energy from an external source, the amplitude of the swing will decrease as it loses energy to its surroundings. The oscillation is said to be damped.

A forced vibration occurs when a body or system is set into vibration by impulses received from another external vibrating source (the driver).

Example:

A tuning fork uses a sound box to amplify the sound it produces.

Vibrations from the tuning fork cause the sound box to vibrate at the same frequency.

Since the sound box has a much larger surface area than the tuning fork, energy is more readily transferred to surrounding air molecules.

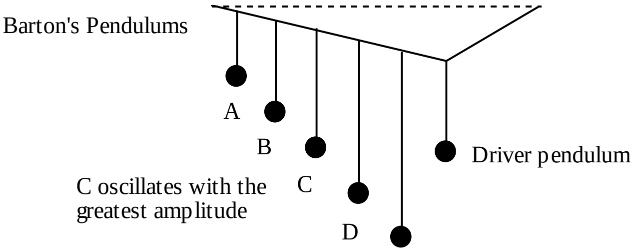
All systems have a **natural frequency** at which they prefer to vibrate.

The amplitude of the forced vibration depends upon the natural frequency of the system compared to the frequency of the driver .

The closer these frequencies, the greater the amplitude of the forced vibration.

We may demonstrate resonance using Barton's Pendulums.

When the driver pendulum is set to oscillate, the other pendula will undergo a forced vibration.



However only pendulum B (which is the same **length** as the driver – not height above the ground) has the same natural frequency. Pendulum B is observed to oscillate with the greatest amplitude over a sustained period.

Resonance

When the driver frequency is equal to the natural frequency of the system, resonance is said to occur.

When resonance occurs the transfer of energy between the driver and the system is most efficient and the energy of the system will increase to a maximum.

This can result in the system acquiring a relatively high amplitude of vibration.

Examples of Resonance

1. The push given to a child's swing must be equal to its natural frequency.
2. Glasses may break if the frequency of a sound equals its natural frequency.
3. Soldiers normally break step when crossing a suspension bridge.
4. Collapse of Tacoma bridge due to wind creating a resonance effect.
5. Resonance in strings and open pipes.

Overtones and Harmonics

Oscillating systems (strings, pipes, drums etc.) have one or more modes of vibration (or frequencies) at which they vibrate naturally.

These **resonant frequencies** depend on the physical nature and dimensions of the system - termed boundary conditions .

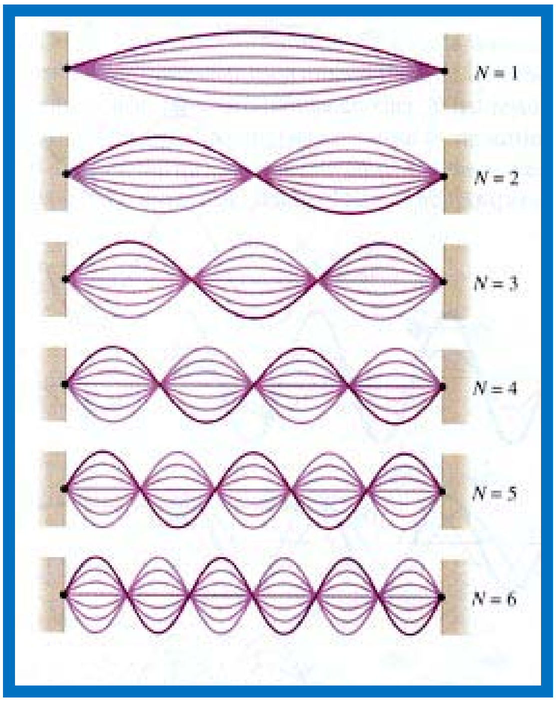
The lowest frequency at which a system will resonate is called the **fundamental frequency**.

At this frequency, it will be vibrating in its **fundamental** or **first mode** of vibration.

The allowed resonant modes of vibration above the fundamental are termed **overtones**.

For a stringed instrument, resonant frequencies occur at integral multiples of the fundamental.

These integral multiples of the fundamental are termed **harmonics**.



Example

1^{st} harmonic = 1 x fundamental frequency

2^{nd} harmonic = 2 x fundamental frequency

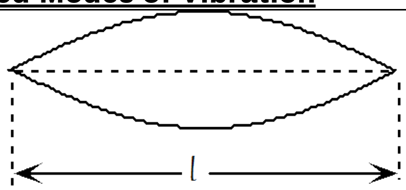
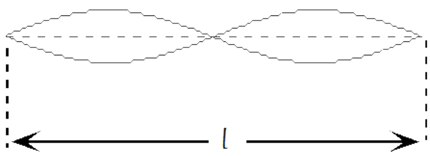
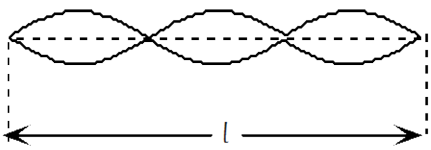
For strings and open pipes, overtones occur at all harmonics.

For closed pipes, overtones occur only at odd harmonics.

Stringed Instruments

When a string on a stringed instrument is plucked progressive waves are generated which reflect from the fixed ends. This causes interference which results in the formation of a standing waves in the string. We may predict the possible modes of vibration and hence the resonant frequencies by applying the boundary condition that a node must exist at each end.

Allowed Modes of Vibration

 	1^{st} Mode of Vibration (Fundamental) Freq of fundamental = f_1 (1^{st} harmonic) $l = \frac{\lambda_1}{2} \quad \therefore \lambda_1 = 2l$ 2nd Mode of Vibration (1^{st} Overtone) $f_2 = 2f_1$ (2^{nd} harmonic) $\lambda = \lambda_2 \quad \therefore \lambda_2 = \frac{2\lambda}{2}$
	3rd Mode of Vibration (2^{nd} Overtone) $f_3 = 3f_1$ (3^{rd} harmonic) $\lambda = \frac{3\lambda_3}{2} \quad \therefore \lambda_3 = \frac{2\lambda}{3}$

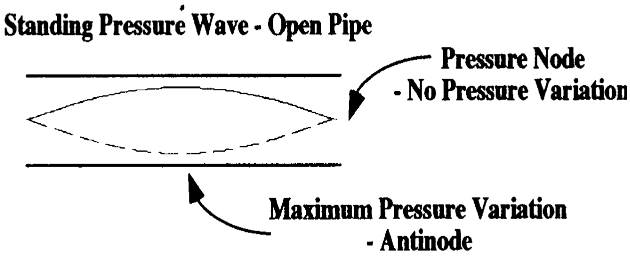
ORGAN PIPES - Resonating Air Columns

If a column of air in a length of pipe is excited, say by blowing over the end or by holding a tuning fork nearby, longitudinal waves travel up and down the pipe being reflected at each end.

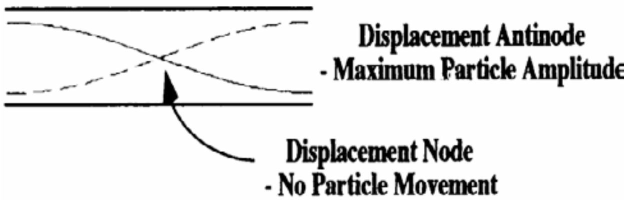
If driven at a resonant frequency of the pipe, standing pressure waves will be formed.

For a pipe open at both ends – called an open pipe (e.g. a flute) pressure waves are reflected 180° (π) out of phase with the incident waves resulting in a pressure node at each end.

Maximum pressure variation occurs between the nodes. These points are pressure antinodes.



If we choose instead to consider the graph of particle displacement we find maximum particle vibration occurs at the open ends with zero displacement mid-way between.

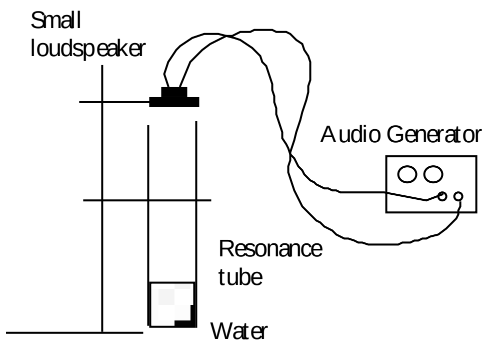


Standing waves may be represented in either displacement or pressure format. The two patterns are 90° "out of phase". From now on we will only use the particle displacement format.

An example of a pipe open at one end but closed at the other (called a closed pipe) is a **resonance tube.**

There are 2 ways of setting this up -

1. Use a varying length of pipe excited by a constant frequency.



Water in a tube can be used to adjust the length of an air column.

When a small loudspeaker is located above the tube, resonance is observed to occur for different lengths of the tube.

Sound waves travel up and down the pipe and are reflected at its ends. This results in the formation of standing waves within the tube.

If we consider the displacement of particles in the tube then for resonance to occur, it is required that a node form at the closed end and an antinode form at the open end.

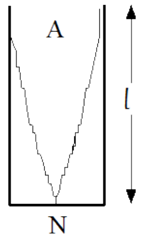
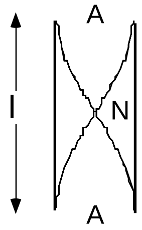
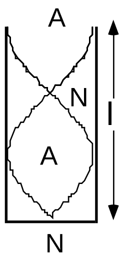
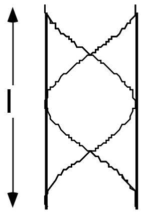
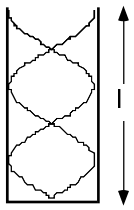
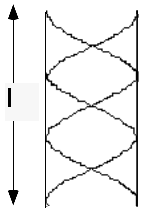
These are the **boundary conditions**.

2. Use a constant length of pipe and excite it by varying the frequency.
If the pipe length is kept a constant length and the driving frequency is changed the tube will resonate at multiples of the fundamental frequency.

The same boundary conditions apply.

Allowed Modes of Vibration (Particle displacement diagrams in a pipe of fixed length)

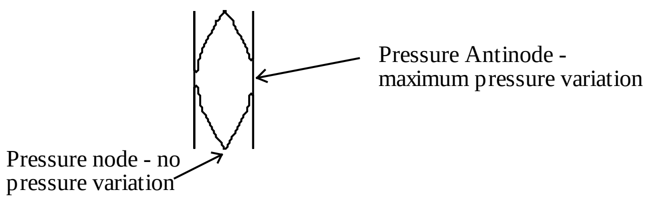
The boundary conditions for a closed pipe are that a particle node forms at one end and a particle antinode forms at the other. An open pipe has a particle antinode at both ends.

Closed Pipes	**Open Pipes**
1st Mode of Vibration (Fundamental) $l = \frac{1}{4}\lambda_1$  frequency = f_1 $\lambda_1 = 4\lambda$	1st Mode of Vibration (Fundamental)  l = $l = \frac{1}{2}\lambda_1$ frequency = f_1 $\lambda_1 = 2\lambda$
2nd Mode of Vibration (1st Overtone) $l = \frac{3}{4}\lambda_2$  $f_2 = 3f_1$ $\lambda_2 = \frac{4\lambda}{3}$	2nd Mode of Vibration (1st Overtone)  $l = \frac{2}{2}\lambda_2 = \lambda_2$ $f_2 = 2f_1$ $\lambda_2 = \lambda = \frac{2\lambda}{2}$
3rd Mode of Vibration (2nd Overtone) $l = \frac{5}{4}\lambda_3$  $f_3 = 5f_o$ $\lambda_3 = \frac{4\lambda}{5}$	3rd Mode of Vibration (2nd Overtone)  l = $3\lambda_3/$ $l = \frac{3}{2}\lambda_3$ $f_3 = 3f_1$ $\lambda_3 = \frac{2\lambda}{3}$

Pressure Variations

Pressure variations that occur within the tube are 90^0 out of phase with the graphs of particle displacement, thus pressure antinodes form at displacement nodes and pressure nodes form at displacement antinodes.

Standing Pressure Wave in an Open Pipe



Exercise Set 6: Resonance in Strings and Air Columns

Velocity of sound in air = 340 ms^{-1}

1. A piano string is tensioned such that the speed of a wave in the string is 102.4 ms^{-1}.
   a) Given that the length of the string is 20.0 cm, find its fundamental frequency
   b) Draw a standing wave pattern for the fundamental vibration.

2. A double bass has strings of length 1.30 m.
   a) Given that the speed of the waves in one string is 2.00×10^2 ms^{-1}, find the first three harmonics for this particular string.

3. Calculate the ratio of the fundamental, first overtone and second overtone frequencies for:
   a) a violin string
   b) an air column open at both ends
   c) an air column open at one end only.

4. An organ pipe is open at both ends.
   a) Draw standing wave diagrams to represent the first three harmonics
   b) Determine the ratio of the wavelengths of these harmonics to the wavelength of the fundamental.

5. A resonating organ pipe 10.0 m long is open at both ends.
   a) Calculate the wavelength of the fundamental frequency.
   b) Using the speed of sound in air as given, calculate the fundamental frequency.
   c) Repeat a and b for the second harmonic of the pipe.

6. A whistle has a distance of 10.0 cm between its opening and its closed end. What is the lowest frequency which can be made by blowing the whistle?

7. A musician designs a single frequency whistle (open at both ends) to play only middle C (256 Hz). What must be the length of the whistle?

8. An organ pipe is designed to play either open at both ends or closed at one end.
   a) Draw a standing wave pattern of the first 3 harmonics for each case.
   b) Calculate the ratio of the wavelengths of the first 3 harmonics for open and for closed pipes.
   c) Calculate the ratio of the frequencies.
   d) What is the difference in the sound of the open and closed pipe in each case?

Answers

1. 256 Hz
2. 77 Hz, 154 Hz, 231 Hz
3. a) 1:2:3 b) 1:2:3 c) 1:3:5
4. b) 6 : 3 : 2
5. a) 20 m b) 17.0 Hz c) 10m, 34 Hz
6. 850 Hz
7. 66.4 cm
8. b) open: 6:3:2, closed: 15:5:3 c) open: 1:2:3, closed: 1:3:5